STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 19

Unit 6 : Introduction to Einstein's Special Theory of relativity

Unit 6: Lorentz transformations.

Introduction to
Special Theory of Relativity

Learning goals:

Discover that the finiteness of the speed of light and
its constant value in all inertial frames of reference requires us to alter our perception of ‘simultaneity’.

This leads to the notion of length-contraction and time-dilation.
Understand how Lorentz transformations account for these.

Furthermore:

We shall learn about the famous 'twin paradox' and how to resolve it….

….. and also about some other fascinating consequences of the STR……

….. Electromagnetic field equations, GTR, GPS clocks, …..

2010 Camaro vs. 2010 Mustang


152

Galilean Relativity


~1650 Kms/hr

In Galilean Relativity:

- The laws of mechanics are the same in all inertial frames of reference.

- The principle of causality/determinism involve the *same interactions* resulting in the *same effects* seen by observers in all inertial frames of references.

- Time t is the same in all inertial frames of references.

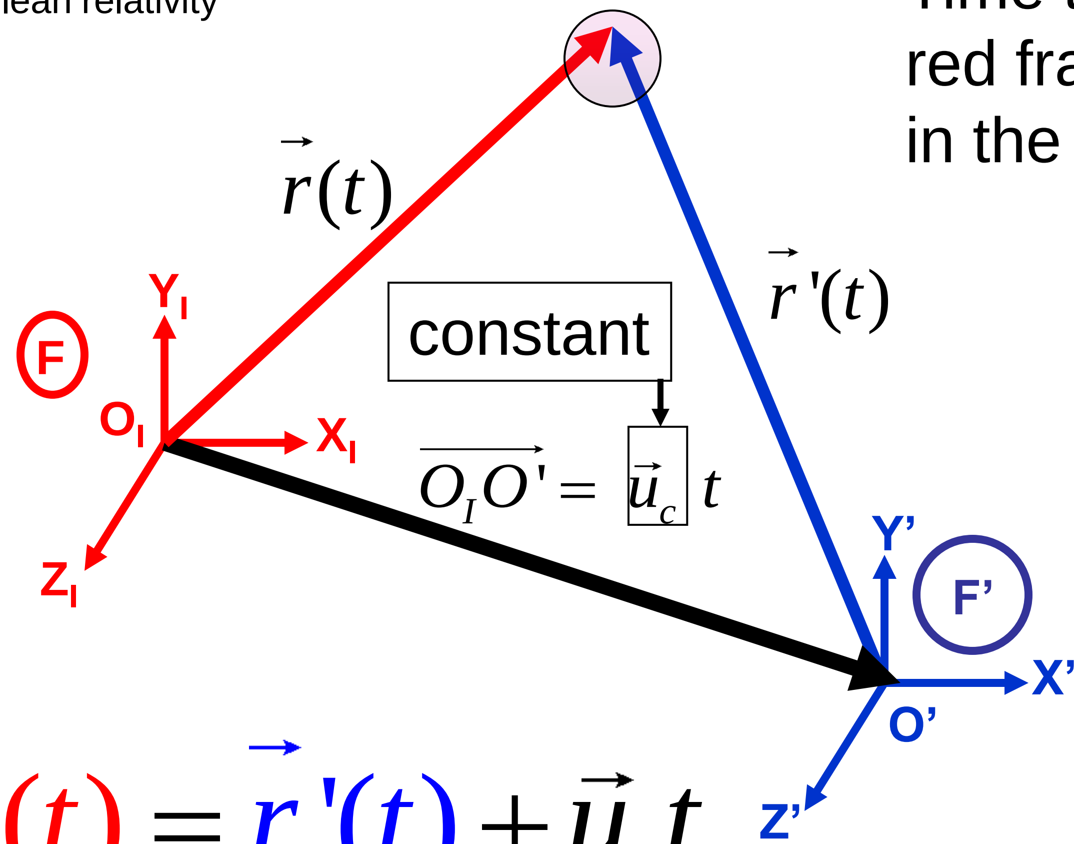
What is the velocity of the oncoming car?

… relative to whom?

Why did the chicken cross the road?

The chicken could be wondering why it is the road that crossed her!

Time t is the same in the red frame and in the blue frame.



$$\vec{r}(t) = \vec{r}\,'(t) + \vec{u}_c t$$

$$\frac{d\vec{r}}{dt} - \vec{u}_c = \frac{d\vec{r}\,'}{dt}$$

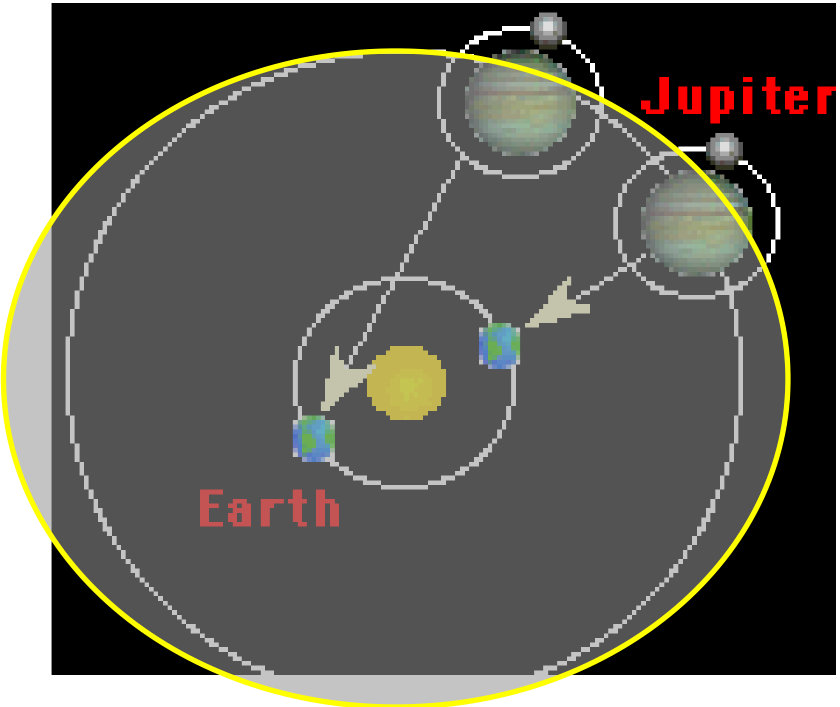
$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \vec{u}_c$$

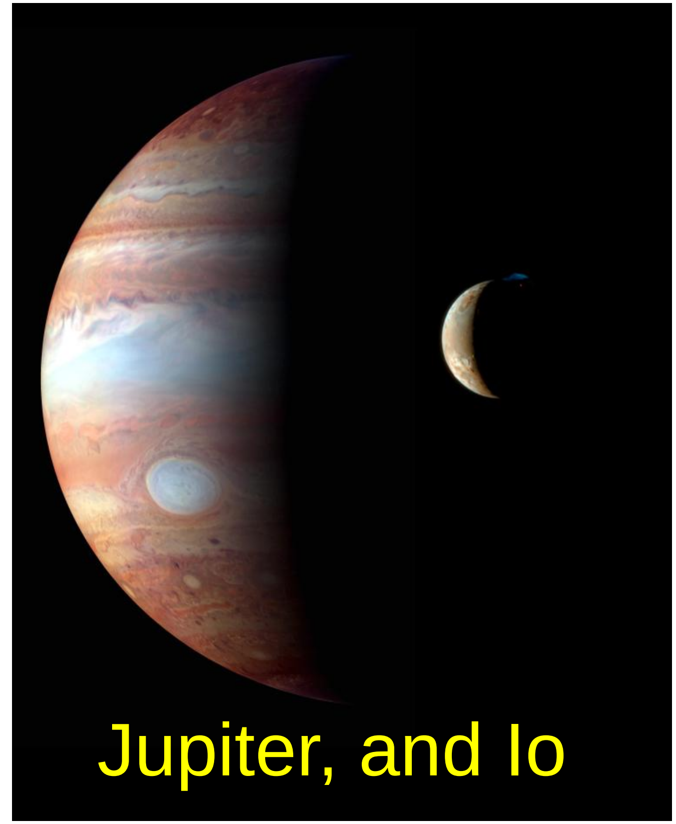
What would happen if the object of your observations is light?

$$\frac{d\vec{r}}{dt} - \vec{u}_c = \frac{d\vec{r}\,'}{dt}$$

Speed of light ?

Danish astronomer Ole Roemer (1644–1710)



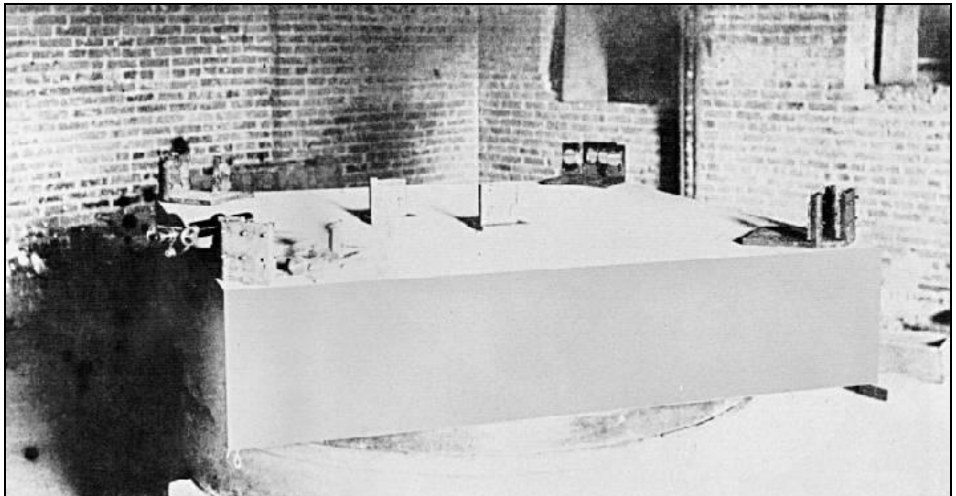


Roemer observed (1675-1676) the timing of the eclipses of Jupiter's moon Io.

Christian Huygens used Roemer's data to calculate the speed of light and found it to be large, but finite!

Light (EM waves) travels at the constant speed in all inertial frames of references.

Experimental proof: *A. A. Michelson and E. W. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," American Journal of Science, 34, 333-345 (1887).*



The actual apparatus that was used in the Michelson-Morley experiment

Michelson and Morley mounted their apparatus on a stone block floating in a pool of mercury, and rotated it to seek changes in relation to the motion of the earth in its orbit around the sun. They arranged one set of light beams to travel parallel to the direction of the earth's motion through space, another set to travel crosswise to the motion.

http://www.aip.org/history/einstein/ae20.htm

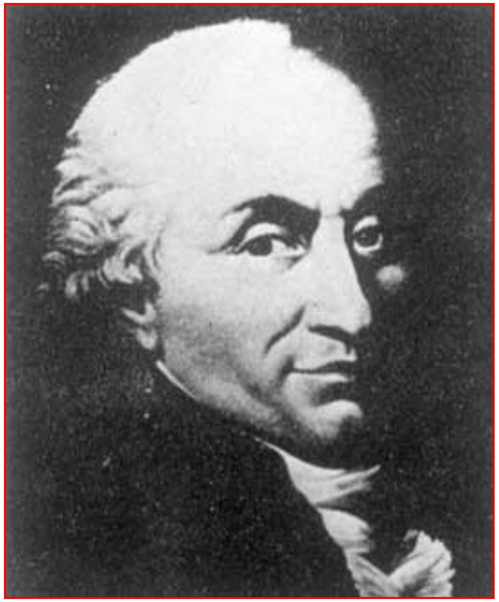
It is debatable whether Einstein paid heed to this particular experiment, but his work provided an explanation of the unexpected result through a new analysis of space and time. http://www.aip.org/history/einstein/emc1.htm

REVIEWS OF MODERN PHYSICS, VOLUME 80, APRIL–JUNE 2008

CODATA recommended values of the fundamental physical constants: 2006*

Peter J. Mohr,† Barry N. Taylor,‡ and David B. Newell§

c = 299 792 458 m s-1



Charles Coulomb
1736-1806

Andre Marie
Ampere
1775-1836

Michael Faraday
1791-1867

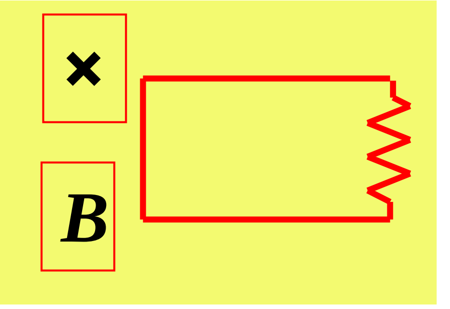
….. other

developments in

Physics ….

Carl Freidrich
Gauss
1777-1855

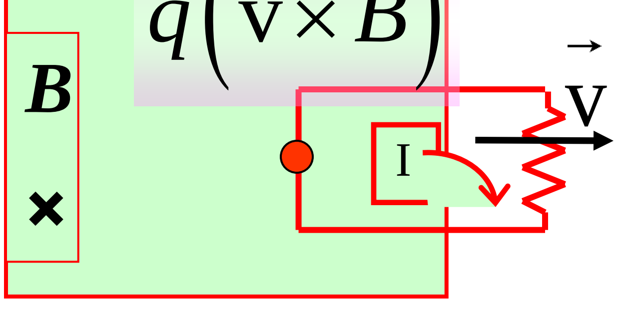
Loop : Stationary



Lorentz force predicts:

(a) Clockwise Current
(b) Counterclockwise Current
√ (c) No Current

Loop : Dragged to the right.



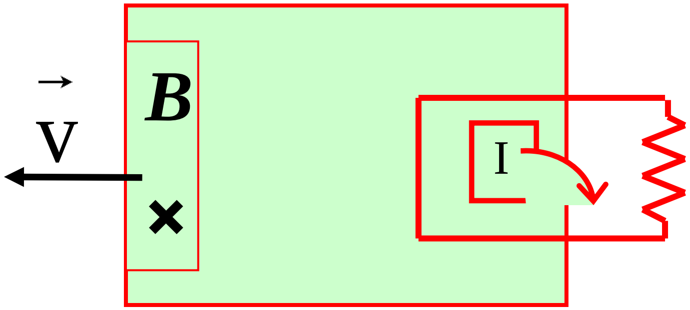
$$q\left(\vec{v}\times\vec{B}\right)$$

$\vec{v}$

Lorentz force predicts:

√ (a) Clockwise Current
(b) Counterclockwise Current
(c) No Current

Faraday's experiments

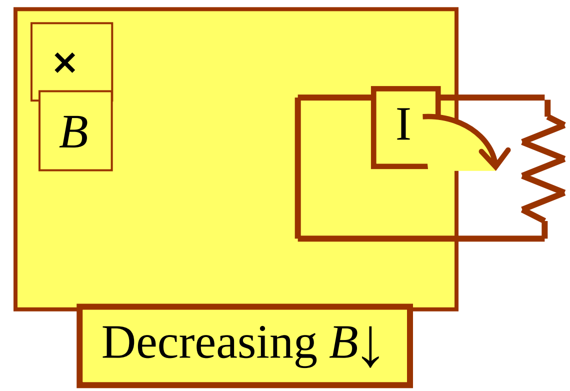


Loop held fixed; Magnetic field dragged toward left.

NO Lorentz force $q\left(\vec{v}\times\vec{B}\right)$

Current: identical!

Strength of B *decreased*.
Nothing is moving,
but still, current seen!!!



$$I \propto \frac{dB}{dt}$$

Einstein:

Special Theory of Relativity

The equations of James Clerk Maxwell

$$\vec{\nabla} \bullet \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Changing magnetic field produces a rotational electric field.

$$\vec{\nabla} \bullet \vec{B} = 0$$

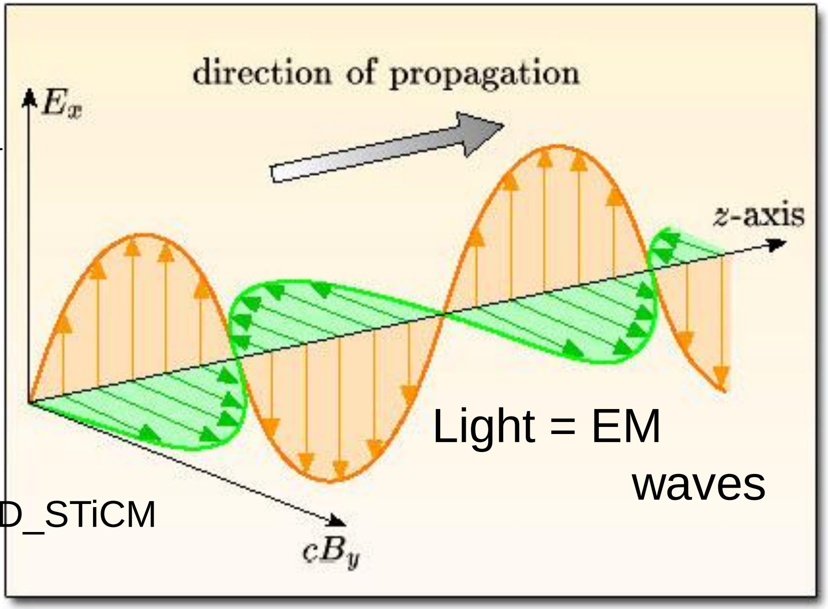
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Changing electric field produces a rotational magnetic field.

$$v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c = 2.9979 \times 10^8 \, m/s$$

Maxwell observed that v obtained as above agreed with the speed of light.

Maxwell's conclusion: "light is an electromagnetic disturbance propagated through the field according to electromagnetic laws"



Speed of light: does not change…

…from one inertial frame of reference to another……

$$\mathrm{v} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = c$$

STR ⟷ ED

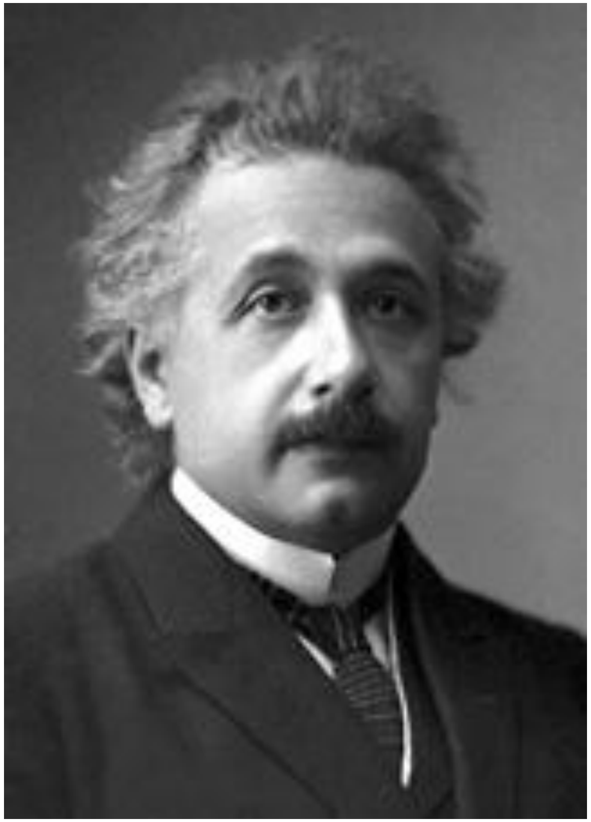
… it is 'time' and 'length' that change!

Electrodynamics & STR

The special theory of relativity is intimately linked to the general field of electrodynamics.

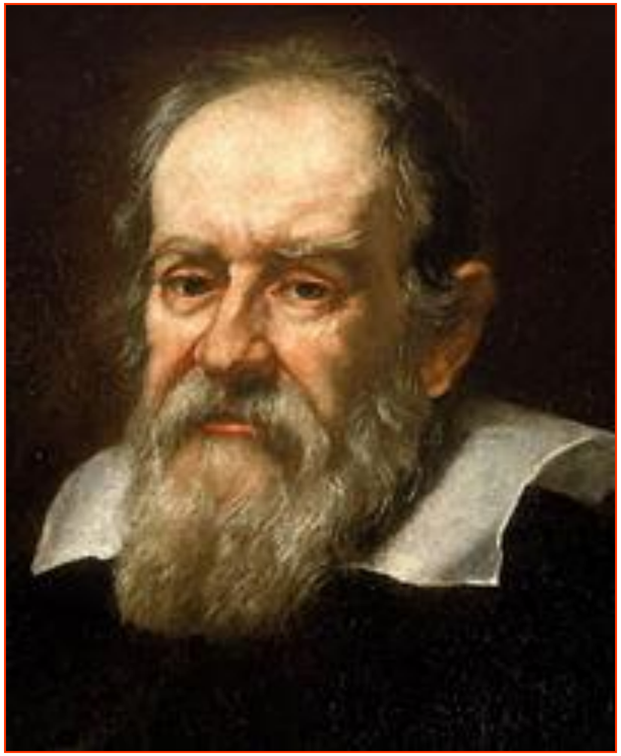
Both of these topics belong to ‘Classical Mechanics’.

James Clerk Maxwell
1831-1879



Albert Einstein
1879 - 1955

Galilean & Lorentz Transformations. Special Theory of Relativity.



Galileo Galilei
1564 - 1642

Hendrik Antoon Lorentz
1853-1928

Smoking is injurious to health!

Albert Einstein
1879-1955

Albert Einstein's ANNUS MIRABILIS 1905

(i) Brownian motion: established the study of fluctuation phenomena as a new branch of physics….. … statistical thermodynamics, later developed by Szilard and others, and for a general theory of stochastic processes.

(ii) Photoelectric Effect – the work that was cited explicitly in Einstein's Nobel Prize!

(iii) Special Theory of Relativity

STR Upshots:

- Physical laws are the same in all inertial reference systems.

- Speed of light in a vacuum is a universal constant for all observers regardless of the motion of the observer or of the source of light.

STR Upshots:

- Max velocity attainable is that of light.
- Objects appear to contract in the direction of motion;
- Rate of moving clock seems to decrease as its velocity increases.
- Mass and energy are equivalent and interchangeable.

Einstein’s theory of relativity:

1905: Special Theory of Relativity

1915, 16: General Theory of Relativity

STR is a ‘special’ case of GTR.

In STR, we compare 'physics' seen by observers in two frames of references moving at constant velocity $\vec{v}$ with respect to each other.

1. Maxwell's equations are correct in all inertial frames of references.

2. Maxwell's formulation predicts : EM waves travel at the speed $c = \sqrt{\dfrac{1}{\mu_0 \varepsilon_0}}$.

3. HENCE, light (EM waves) travels at the constant speed in all inertial frames of references.

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

Notion of TIME itself would need to change

Einstein was clever enough, & bold enough, to stipulate just that!

What happens to our notion of space ?

$$speed = \frac{\text{distance}}{time}$$

We will take a Break…

…… Any questions ?

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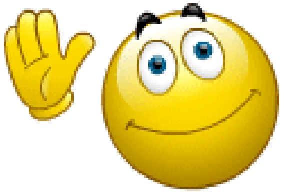

Next L20 : STR

The way we think about space and time must change; it must take into account our motion with respect to each other, even if it is at a constant velocity.

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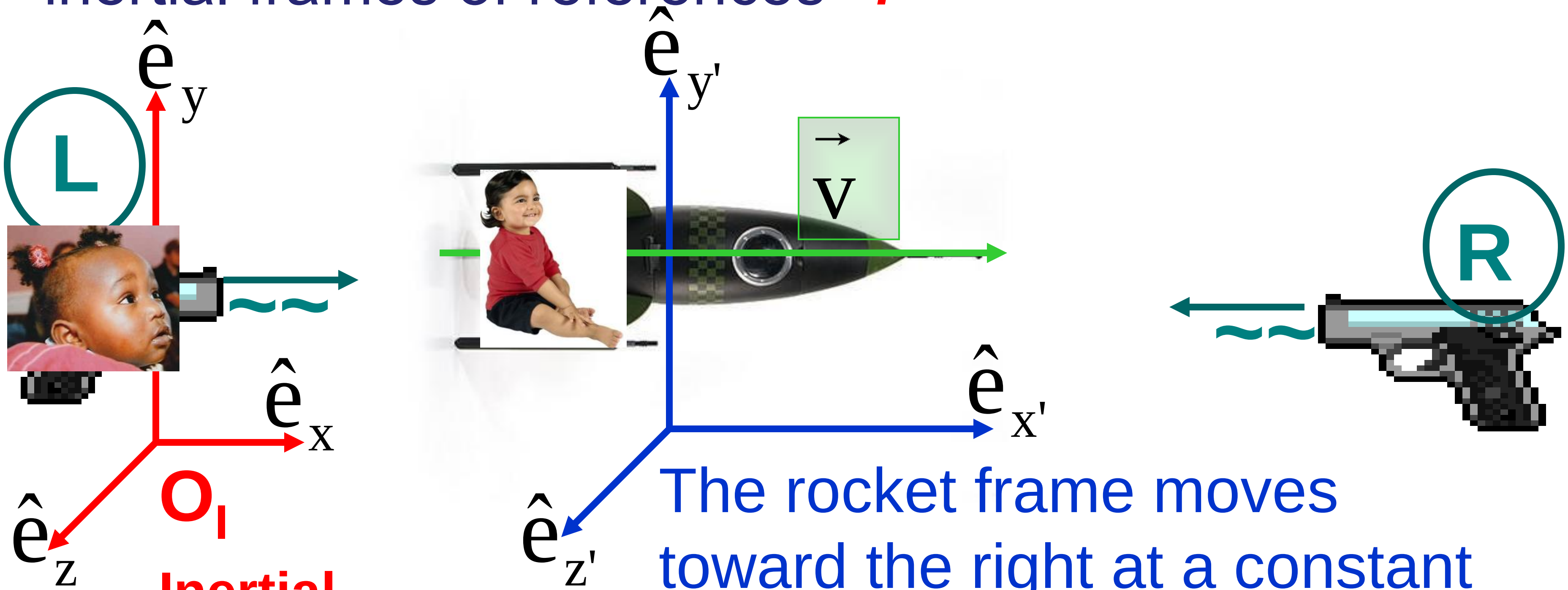
STiCM Lecture 20

Unit 6 : Special Theory of Relativity

Reconciliation with the constancy of the speed of light

Just what does it mean to say that "Light (EM waves) travels at the constant speed in all inertial frames of references" ?



The rocket frame moves toward the right at a constant velocity fc where 0<f<1.

COUNTER-INTUITIVE ?

Speed of light in a vacuum is a universal constant for all observers regardless of the motion of the observer or of the light source

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

MEASUREMENTS

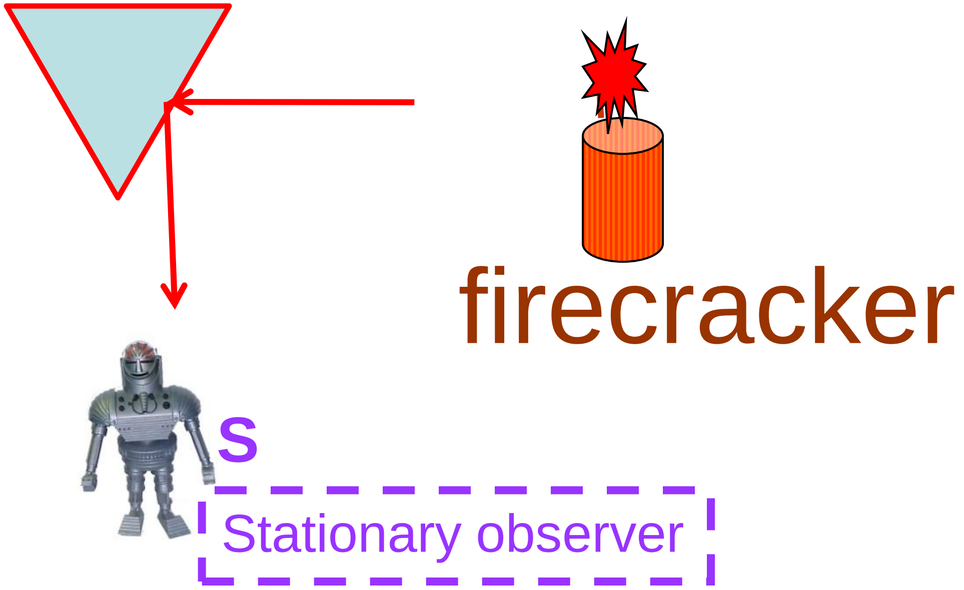
Event: A physical event/activity that takes place at (x,y,z) at the instant t.

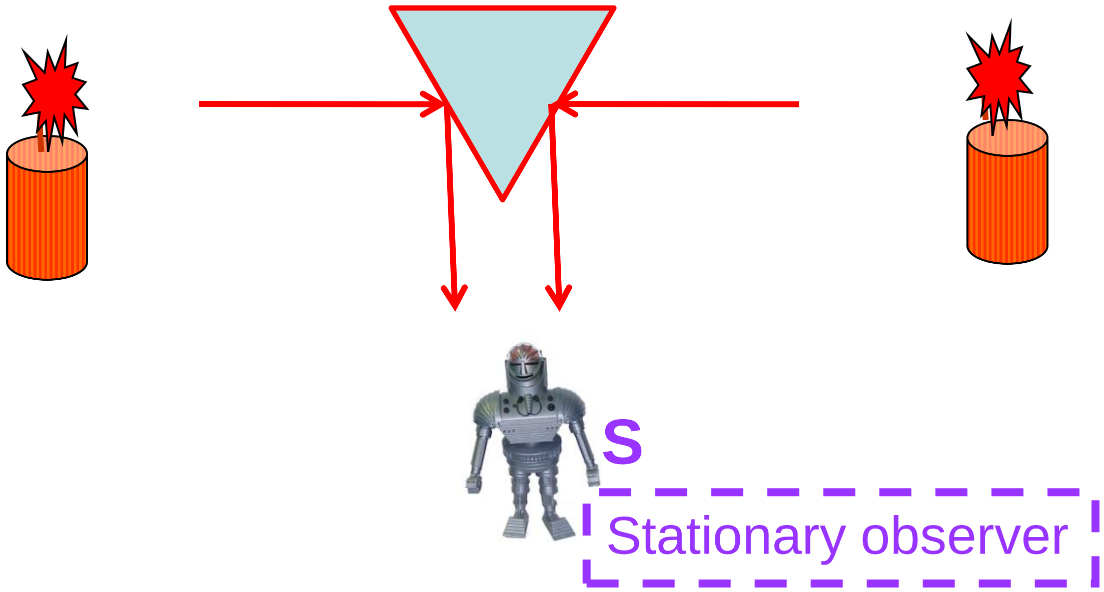
SPACE-TIME COORDINATES of the EVENT: (x,y,z,t), in a frame of reference S.

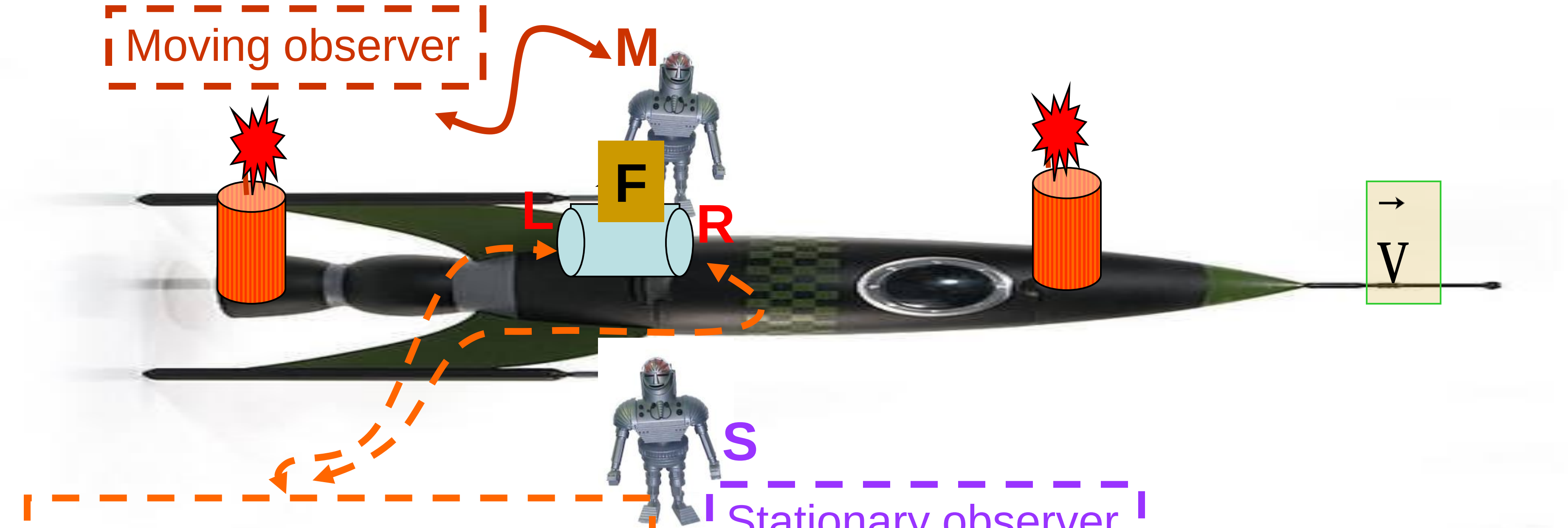
In another frame S', the coordinates are: (x',y',z',t').

We must revise our notions of 'simultaneity'.

Events that are 'simultaneous' in one frame of reference S are *not* so in another frame of reference S' that is moving relative to S.

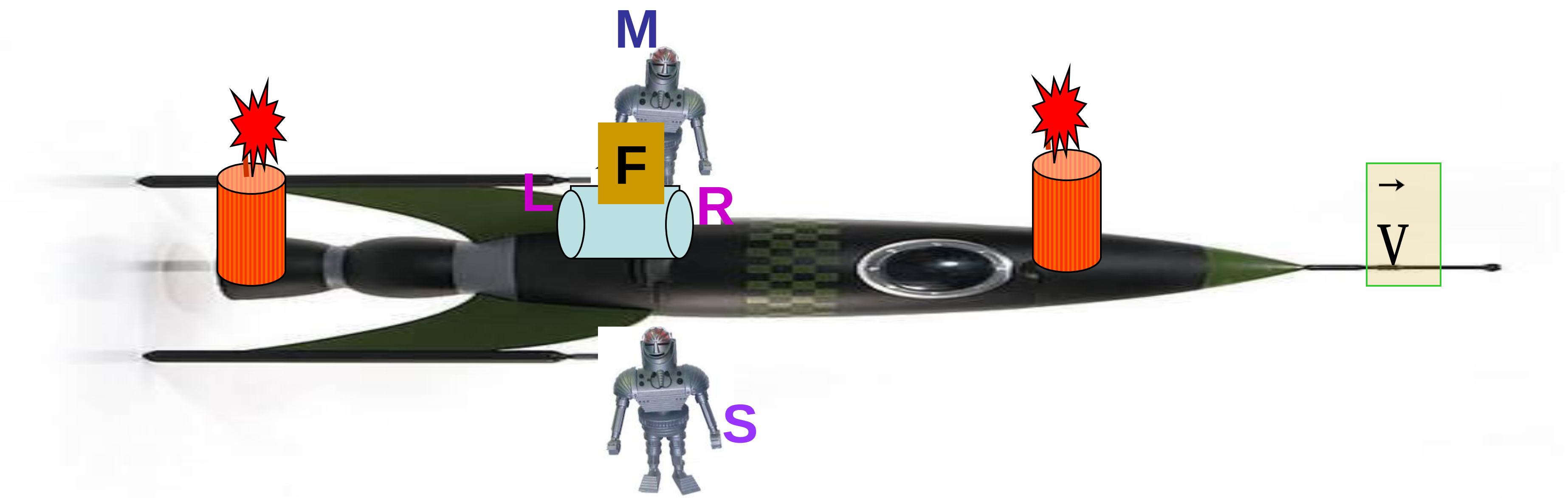

firecracker
S
Stationary observer


S
Stationary observer



Light sensors would tell observer M the sequence at which light from left-cracker and right-cracker reach the tiny, infinitesimal sensor.

The two firecrackers explode 'simultaneously' as seen by S just as M crosses S.



(1)S detects both the flashes simultaneously.

(2)Light from both explosions travels at equal speed toward S.

(3) M would expect his sensor to record light from the right-cracker, *before* it senses light from the one on our left side.

Events that seem SIMULTANEOUS

to the stationary observer do not seem

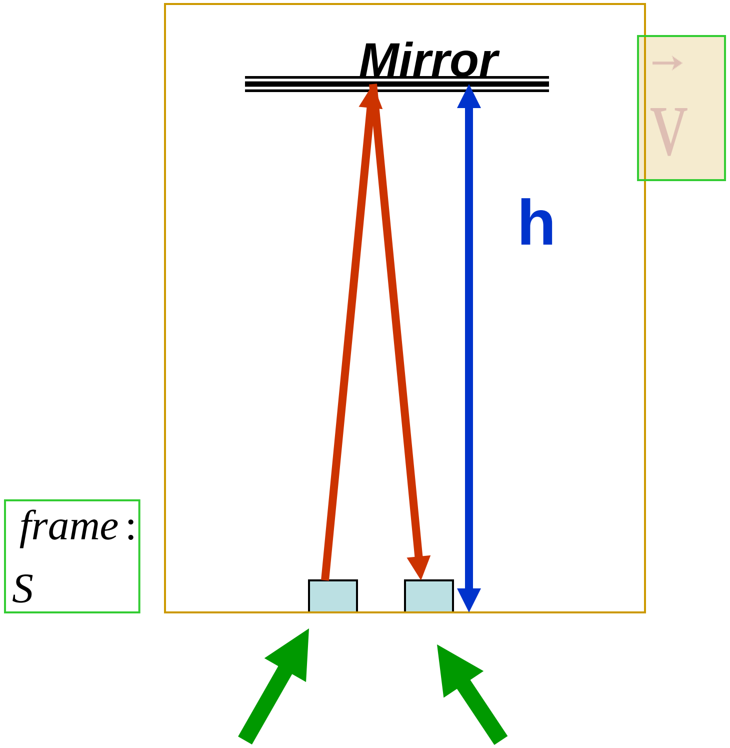
to be so to the moving observer – who

also is in an inertial frame *!*

So, let us, in all humility, reconsider our notion of TIME and SPACE !

First, we examine how a clock clocks TIME.

'LIGHT CLOCK' 'TIME KEEPING COUNTER'



Light Source Light Detector

Both of these: infinitesimal size

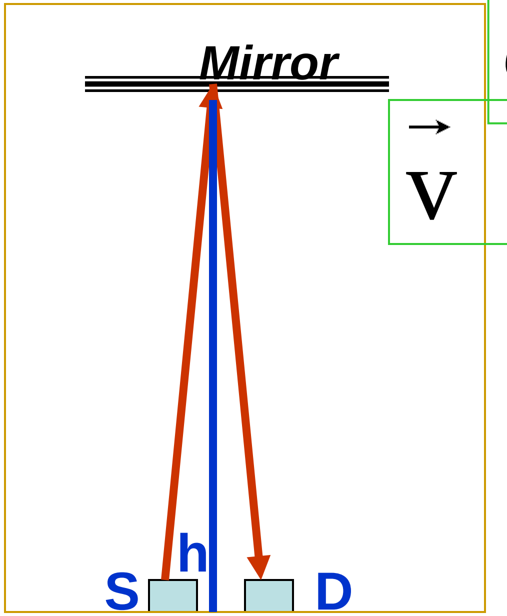
gedanken experiment

The clock advances by one tick every time the detector receives a pulse.

Furthermore, as soon as the detector receives the pulse, the source gets triggered to emit the next pulse.

The Light Clock moves at velocity $\vec{V}$ in a frame of reference S.
In the clock-frame S', the Light Clock is of course at rest.

$clock\ frame,\ S'$



$frame: S$

$$Clearly,\ \Delta t' = t'_2 - t'_1 = 2h/c$$

As the pulse travels over an interval Δt
from source to mirror, to detector,
in the frame S,
the light-clock itself advances to the right
through a sideways distance of vΔt.

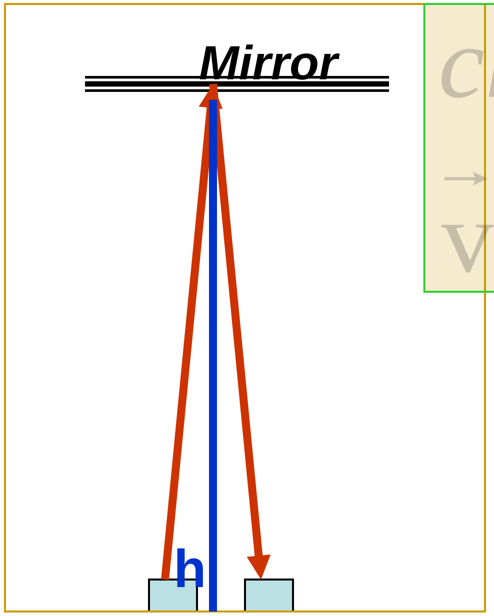
$$\left[\left(\frac{1}{2}\Delta t\right)c\right]^2 = h^2 + \left[\left(\frac{1}{2}\Delta t\right)v\right]^2$$

$$\Rightarrow \Delta t = \frac{2h/c}{\sqrt{1-v^2/c^2}} = \frac{\Delta t'}{\sqrt{1-\beta^2}}$$

$where\ \beta = v/c.$

In frame S, the light pulse passes along the oblique direction, a distance $= \left(\frac{1}{2}\Delta t\right)c,$ *from S to M, and equal* oblique distance *from M to D.*

If β>1, Δt would become imaginary. That would be absurd!
To prevent that, v<c always. c is *not* reachable by anything.



frame : S

clock frame, S'

$\vec{v}$

$$\Delta t' = t_2' - t_1' = 2h/c = \Delta\tau$$

$$\Delta t = \frac{2h/c}{\sqrt{1-v^2/c^2}} = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$$

$$\beta = v/c \langle 1$$

$$\Delta\tau = \Delta t' = PROPER\ TIME$$

$$\text{distance} = time \times speed$$

$$\left[\left(\frac{1}{2}\Delta t\right)c\right]^2 = h^2 + \left[\left(\frac{1}{2}\Delta t\right)v\right]^2$$

$$\Delta t > \Delta\tau$$

Time Dilation

$$\Rightarrow \Delta t = \frac{2h/c}{\sqrt{1-v^2/c^2}} = \frac{\Delta t'}{\sqrt{1-\beta^2}}$$

where $\beta = v/c.$

Conclusions do not depend on the use of the ‘Light Clock’. Any clock would give the same result.

If one of the twins travels, the home-bound sibling ages more than the travelling one!

$$\Delta t = \frac{2h/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$$

$$\beta = v/c$$

$$\Delta\tau = \Delta t' = PROPER\ TIME$$

$$\Delta t > \Delta\tau$$

Time Dilation

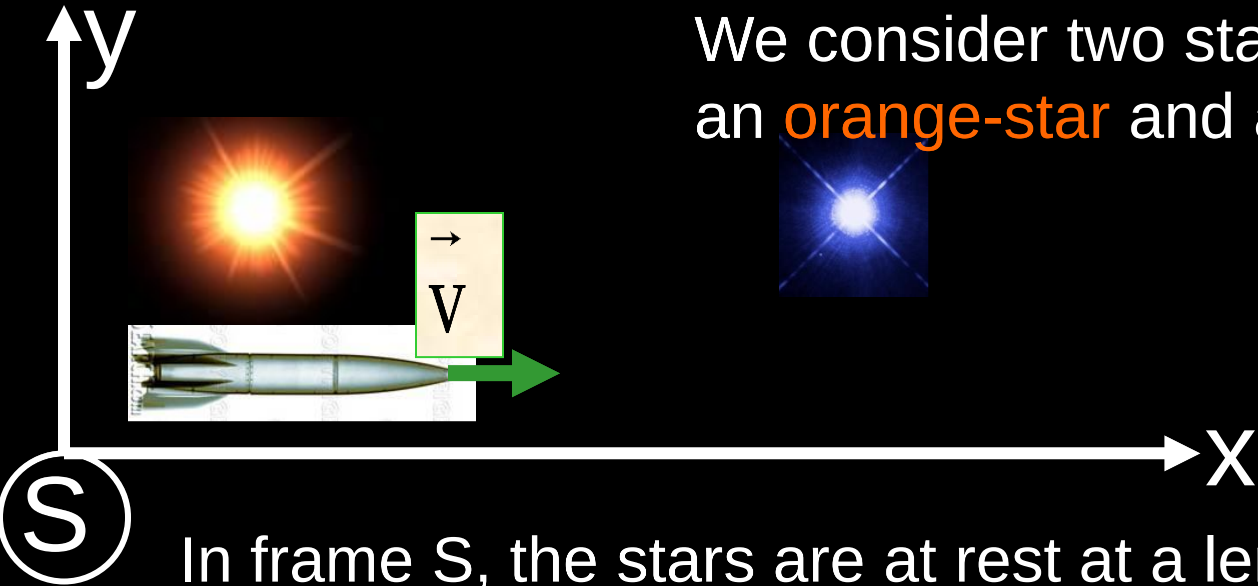
"Moving clocks go slow; time interval between two ticks is longer when measured in a frame in which the clock is moving"

In as much as we have had to modify our notion of time-interval, we are required to modify our notion of space-interval as well.

Thus, we are led not only to Time Dilation but also to Length Contraction.

Both of these modifications become necessary on account of the ‘counter-intuitive’ fact that “Light (EM waves) travels at the constant speed in all inertial frames of references”.

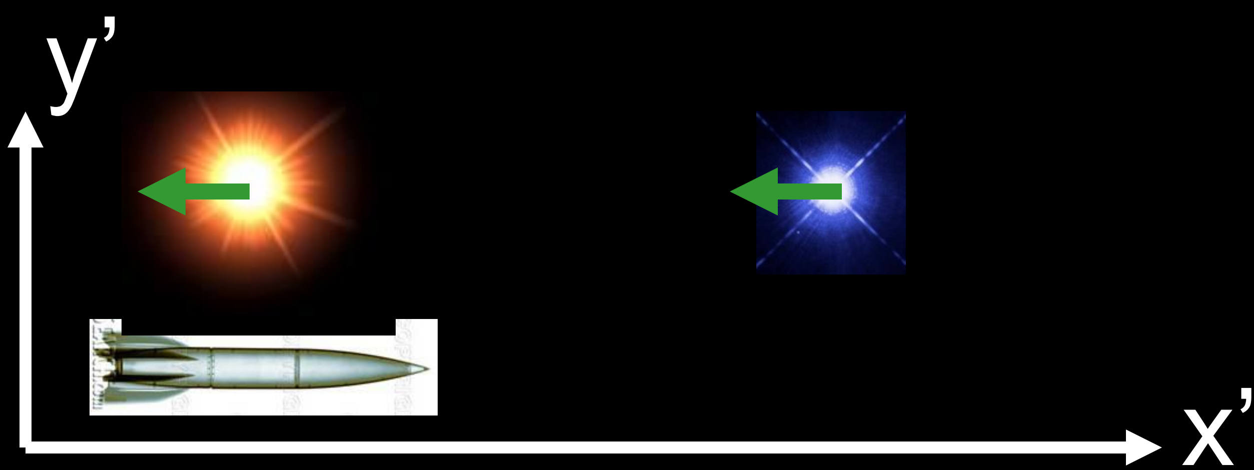
We consider two stars in deeeeeep space, an orange-star and a blue-star.



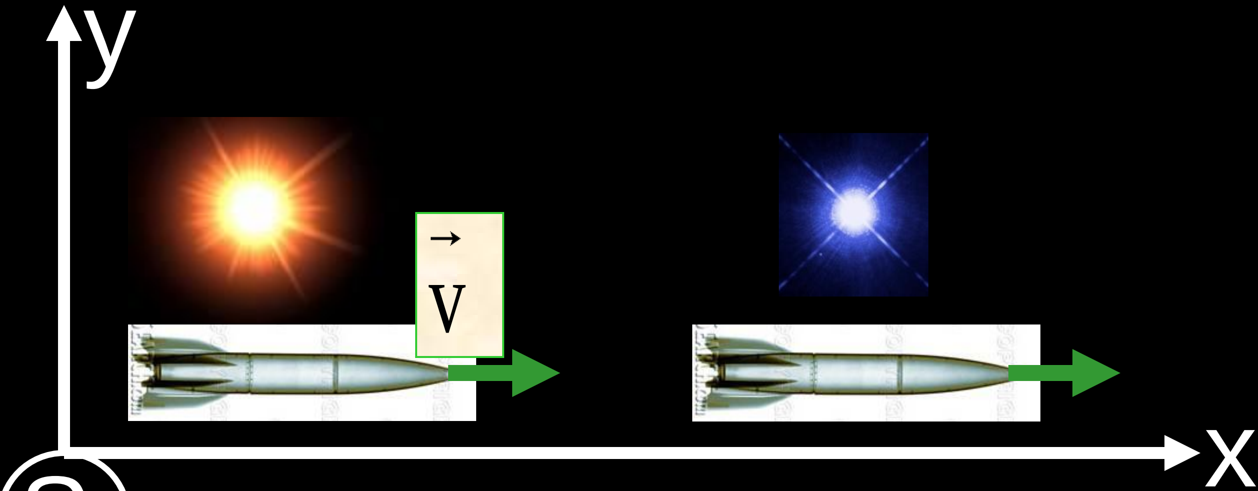
In frame S, the stars are at rest at a length L apart from each other, and a rocket flies by as shown.

An observer in frame S carries out measurements of lengths, and of time intervals.

In the ROCKET-FRAME-S’, the stars **O** and **B** move to the left at the same relative speed.



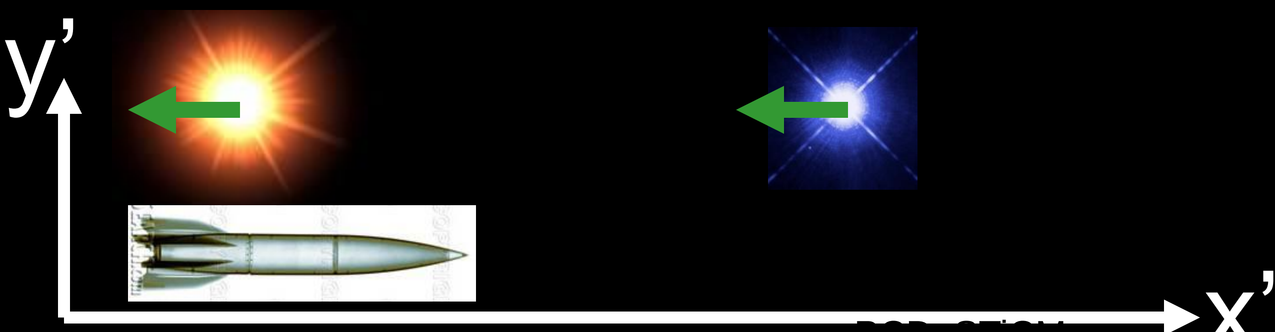
S’: Rocket frame



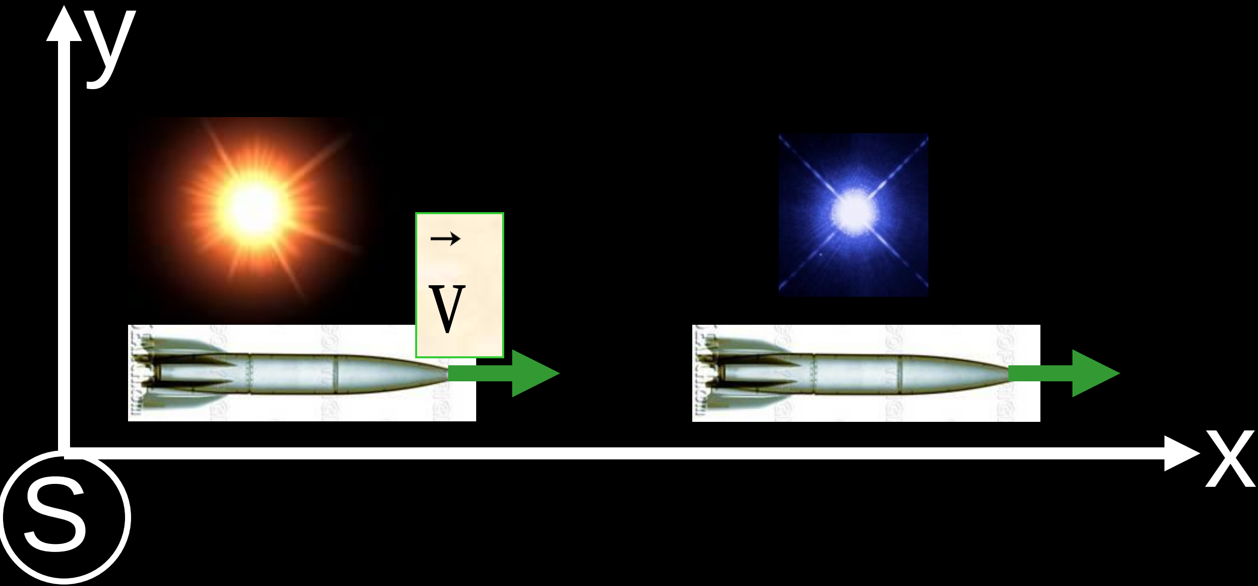
S

In frame S, the objects are at rest at a length L apart from each other.
This LENGTH L is therefore the “PROPER LENGTH”, *l*.

In S’, the clock is at rest in the rocket and yields, therefore, “PROPER TIME”.



S’: Rocket Frame



$$L = \Delta x = x_{orange} - x_{blue}$$

is the LENGTH (distance) *between the two stars in frame S.*

Also, in this frame, the time measured for the journey is Δt.

$$\text{Rockets's speed} = v = \frac{L}{\Delta t} = \frac{l}{\Delta t},$$

where l is the PROPER LENGTH

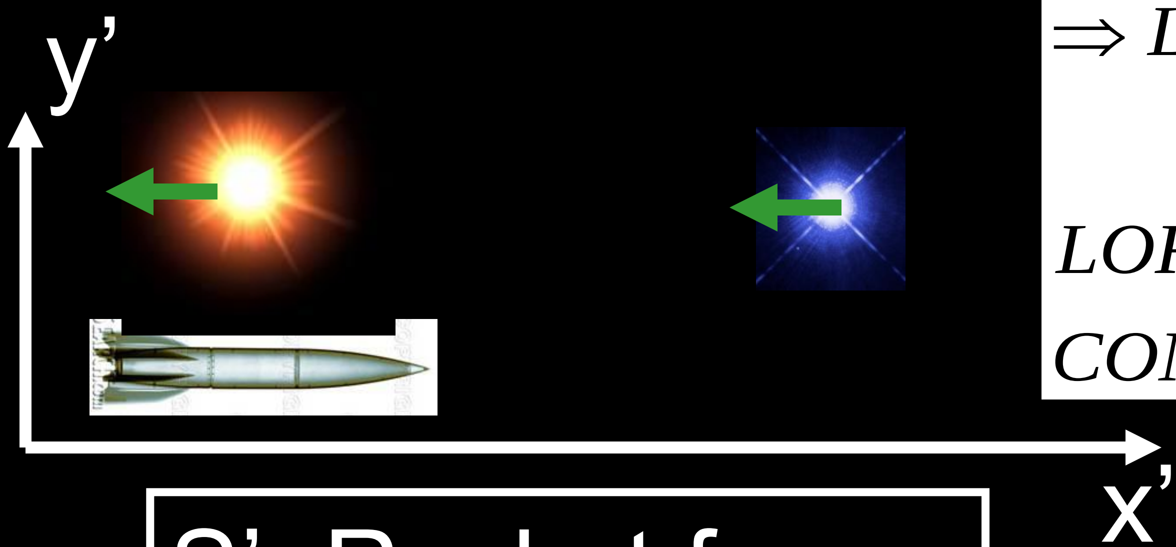
Note! The stars are fixed in space in frame S.

In the ROCKET-FRAME-S', it is the two stars that move to the left at speed $v=\dfrac{L'}{\Delta t'}$, *where* $\Delta t'$ *is the PROPER TIME* $(\Delta\tau)$ *measured in* S' *for the blue star to travel the LENGTH* L'.

$$v = \frac{L'}{\Delta t'} = \frac{L'}{\Delta \tau} = \frac{L'}{\left(\sqrt{1-\beta^2}\right)(\Delta t)}$$

$$\Rightarrow L' = l\sqrt{1-\beta^2} \le l$$

LORENTZ (LENGTH) CONTRACTION



S': Rocket frame

Hendrik Antoon Lorentz
1853-1928

Pieter Zeeman
1865-1943

1902 Nobel Prize in Physics

"in recognition of the extraordinary service they rendered by their researches into the influence of magnetism upon radiation phenomena"

Lorentz contraction!

Lorentz moving up!

Lorentz moving to right!

http://www.bun.kyoto-u.ac.jp/~suchii/lorentz.tr.html

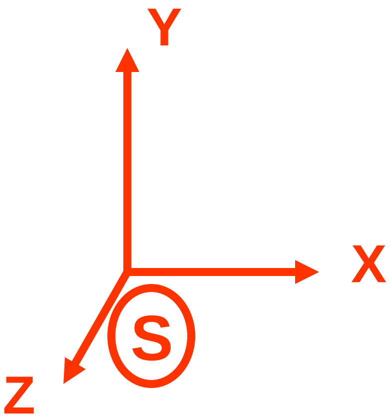
LORENTZ transformations (x,y,z,t) to (x’,y’,z’,t’)

Requirements:

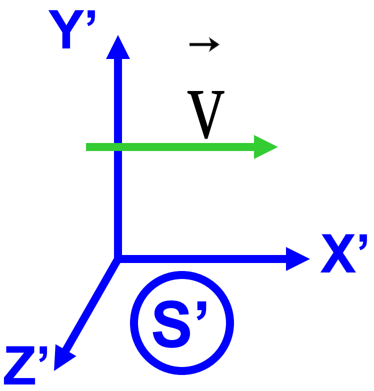
Ensure that speed of light is same in all inertial frames of references.

Transform both space and time coordinates.

Transformation equations must agree with Galilean transformations when v<<<c.



Origins O and O' of the two frames S and S' coincide at t=0 and t'=0.



$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \beta^2}}$$

$Note: \gamma \to 1$ as $v \to 0$.

Lorents transformations transform the space-time coordinates of ONE EVENT.

Main Reference:

‘Physics for Scientists and Engineers’, II Edition,

by

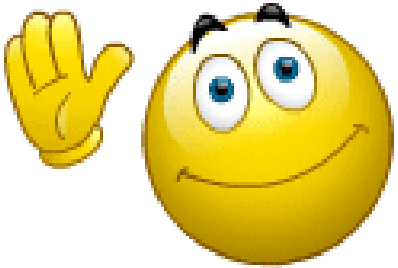
Randall D. Knight

(Pearson, Addison-Wesley, 2007)

Both 'time dilation' and 'length contraction' are automatic consequences of the constancy of speed of light in all inertial frames of references.

We will take a break…

…… Any questions ?



Next L21 : STR

Twin Paradox, etc.

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STiCM Lecture 21

Unit 6 : Special Theory of relativity

Twin Paradox, other STR consequences

"Light (EM waves) travels at the constant speed in all inertial frames of references".

'~~counter-intuitive~~'

'educated intuition'

Both 'time dilation' and 'length contraction' are automatic consequences of the constancy of speed of light in all inertial frames of references.

Consequences:

Length Contraction.

Time Dilation.

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-\beta^2}}; \quad \beta = \mathrm{v/c} \langle 1 ; \quad L' = l\sqrt{1\text{-}\beta^2} \le l$$

- re-interpretation of 'momentum' and 'energy'

Seeta and Geeta are identical twins.

Twin Paradox

Geeta stays at home,

and Seeta travels in a rocket at a speed $\frac{4}{5}c$ for 3 yrs

measured in the rocket-clock (proper time).

Geeta's home-based clock measures

the corresponding time interval as $\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$; $\beta = \text{v/c} = \frac{4}{5}$.

$$\sqrt{1-\frac{\text{v}^2}{c^2}} = \sqrt{1-\frac{\left(\frac{4}{5}c\right)^2}{c^2}} = \frac{3}{5}$$

$$\Delta t = \frac{\Delta\tau}{3/5} = \frac{5}{3}(3yrs) = 5yrs$$

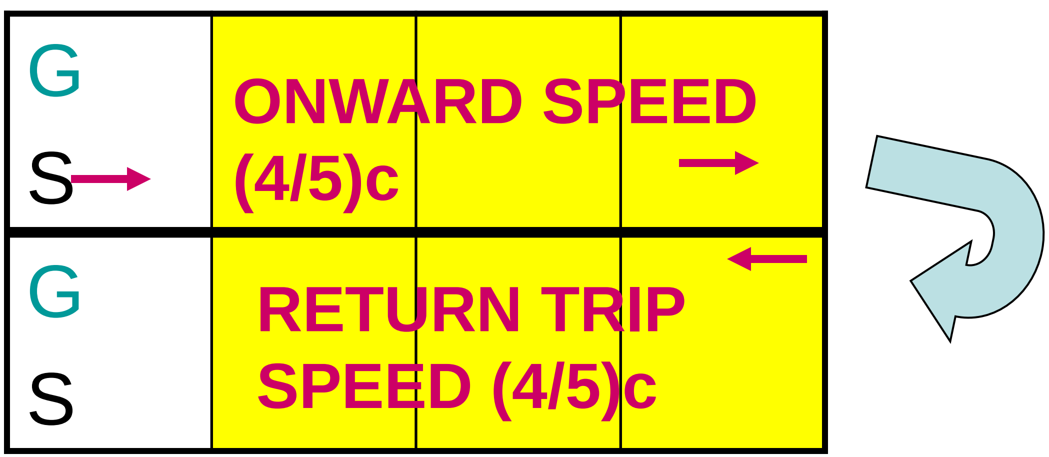
$$\Delta\tau = \Delta t' = PROPER\ TIME$$

$$\Delta t > \Delta\tau \quad (Time\ Dilation).$$

Geeta has aged by 5 years during Seeta's travel over which the latter has aged by only 3 years!

But why should we think this is a paradox? It sure isn't !

Seeta now turns around, and returns at the same speed, thus taking another 3 years (measured, of course, in her clock in the rocket frame) to return, during which Geeta's clock advances by another 5 yrs.



During Seeta's round trip then, home-bound Geeta would age by 10 years, and travelling Seeta by only 6 years!

Even this isn't a paradox – of course!

During Seeta's round trip then, home-bound <u>Geeta would age by 10 years</u>, and travelling Seeta by 6 years.

But,

just what is the paradox?

Symmetry/Equivalence principle in STR: From the point of view of Seeta's perspective, it is Geeta who appears as the traveling sibling, …. and would be therefore younger than Seeta!

The two observers being in equivalent inertial frames, must see same 'physics'

We have a PARADOX !

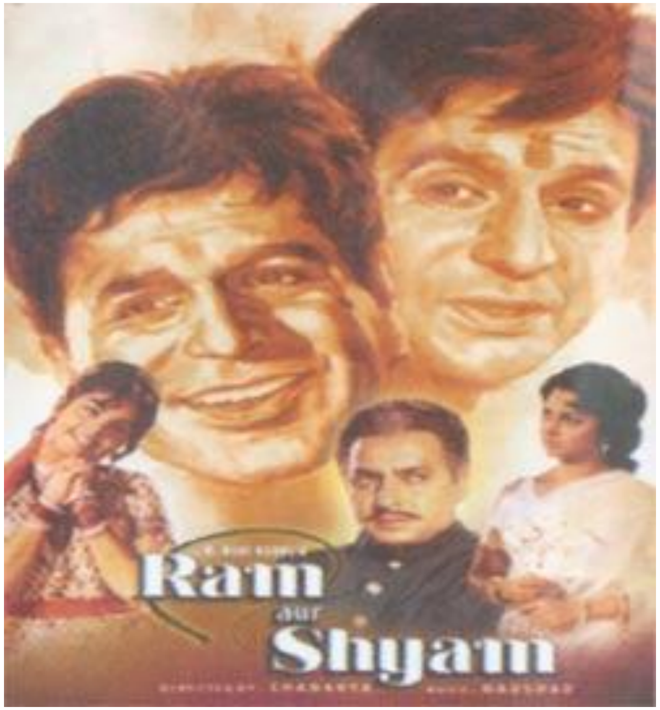
Resolution of the ‘paradox’ would occur if we establish the fact that:

From the points of views of BOTH Geeta and Seeta,

if they looked at their respective clocks,

home-bound Geeta would age by 10 years,

and traveling Seeta by 6.



In some (published) comments on the twin-paradox, resolution has been sought by invoking Seeta's acceleration from the U-turn when she would begin Geeta's chase after 3 years.

Other 'explanations' employ GTR !

However, such 'explanations' are <u>not</u> called for.

We resolve the paradox WITHIN the framework of STR without invoking any acceleration.

One can do away completely with Seeta's acceleration by considering in our thought-experiment a **third observer Jayalalitha.**

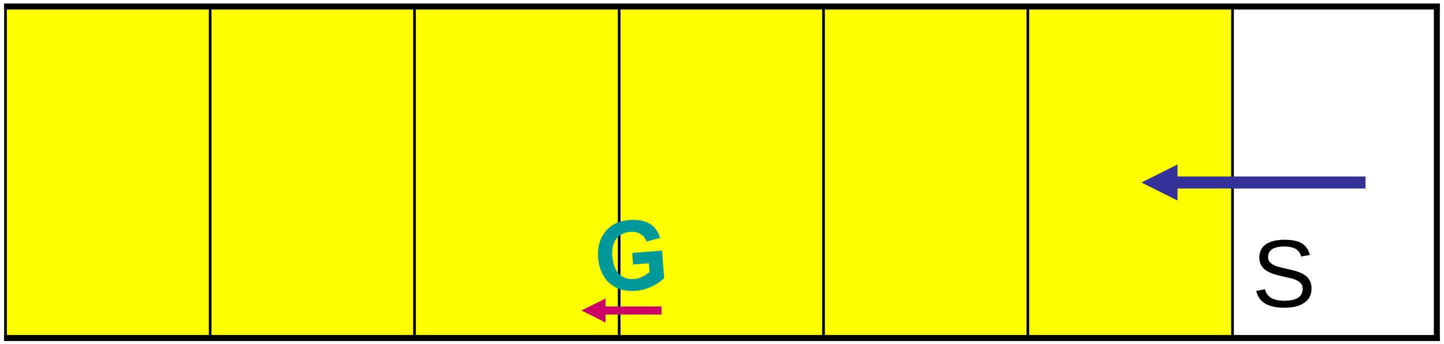
Weren't there a set of triplets, rather than mere twins? This third observer would not undergo any acceleration, but only pass by both Seeta and Geeta and communicate the time-intervals.

Jayalalitha would pass Seeta, and then catch up with Geeta and compare her clock with Geeta's as she crosses her, and then send that information back to Seeta.

The paradox is to be resolved within the framework of STR
– no acceleration of any frame must be invoked.
- GTR is irrelevant here.

Explanation within the framework of STR, and without involving any acceleration of any frame of reference.

SEETA'S PERSPECTIVE.



Geeta takes off (along $-\hat{e}_x$) at 0.8c

Seeta clocks 3 years in her wait, -

- and then take off to catch up with Geeta -

- who continues her travel at the same earlier speed.

Question: At what speed should Seeta travel to catch up with Geeta in 3 <u>additional</u> years as per Seeta's clock?

You & your Dad plan to go for a dinner at a restaurant that is 5 kms away. Your table is booked for 9pm.

Your Dad starts out at 7pm and walks @ 3 Kms/hr for one hour. After the first hour, he gets a bit tired, but needs to walk only @ 2 kms/hr to reach the restaurant at 9pm.

You start out at 8pm, and must meet your Dad at the restaurant at 9pm. What must be your speed?

Sum of the velocities, for Seeta to catch up:

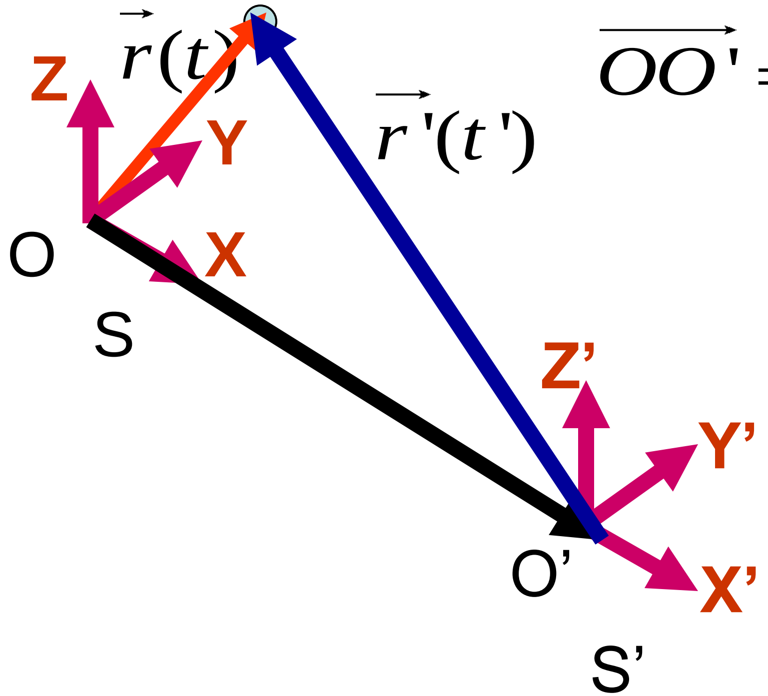
$$\frac{4}{5}c+\frac{4}{5}c=\frac{8}{5}c \;\rangle\rangle\; \mathrm{c}\;!$$

… as per Galilean relativity

Impossible for Seeta to get that speed > c

This is not how relative velocity is added!

One must use Lorentz, not Galilean, relativity.



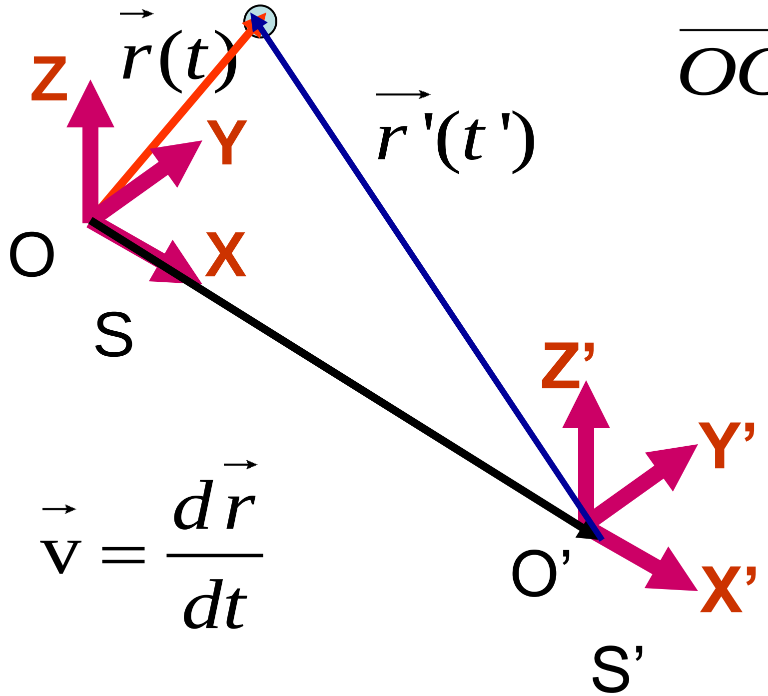
$$\overrightarrow{OO'} = (u_x \hat{e}_x)t$$

$$x' = \gamma(x - u_x t)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{u_x x}{c^2}\right)$$

$$\vec{\mathrm{v}} = \frac{d\vec{r}}{dt} = \frac{\mathrm{dx}}{\mathrm{dt}}\hat{e}_x + \frac{\mathrm{dy}}{\mathrm{dt}}\hat{e}_y + \frac{\mathrm{dz}}{\mathrm{dt}}\hat{e}_z$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\vec{\mathrm{v}}' = \frac{d\vec{r}\,'}{dt'} = \frac{\mathrm{dx'}}{\mathrm{dt'}}\hat{e}_x + \frac{\mathrm{dy'}}{\mathrm{dt'}}\hat{e}_y + \frac{\mathrm{dz'}}{\mathrm{dt'}}\hat{e}_z$$



$$\overrightarrow{OO'} = (u_x \hat{e}_x)t$$

$$x' = \gamma(x - u_x t)$$

$$t' = \gamma\left(t - \frac{u_x x}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\vec{\mathrm{v}} = \frac{d\vec{r}}{dt}$$

$$\vec{\mathrm{v}}\,' = \frac{d\vec{r}\,'}{dt'} = \boxed{\frac{\mathrm{dx'}}{\mathrm{dt'}}}\hat{e}_x + \frac{\mathrm{dy'}}{\mathrm{dt'}}\hat{e}_y + \frac{\mathrm{dz'}}{\mathrm{dt'}}\hat{e}_z$$

$$\frac{\mathrm{dx'}}{\mathrm{dt'}} = \frac{d(\gamma(x - u_x t))}{d\left(\gamma\left(t - \frac{u_x x}{c^2}\right)\right)} = \frac{d(x - u_x t)}{d\left(t - \frac{u_x x}{c^2}\right)} \Rightarrow \frac{\mathrm{dx'}}{\mathrm{dt'}} = \frac{\mathrm{v_x} - u_x}{1 - \frac{u_x \mathrm{v_x}}{c^2}}$$

$$\frac{dx'}{dt'} = \frac{v_x - u_x}{1 - \frac{u_x v_x}{c^2}}$$

If the frame of reference S' is moving in the negative x direction, we shall get:

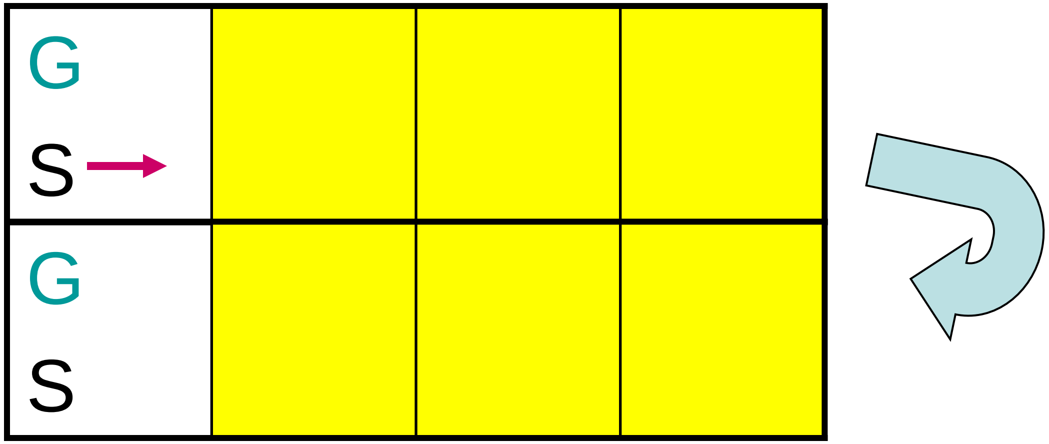
$$\frac{dx'}{dt'} = \frac{v_x + u_x}{1 + \frac{u_x v_x}{c^2}}$$

$$v_x' = \frac{v_x + u_x}{1 + \frac{u_x v_x}{c^2}}$$

$$v_y' = v_y$$

$$v_z' = v_z$$

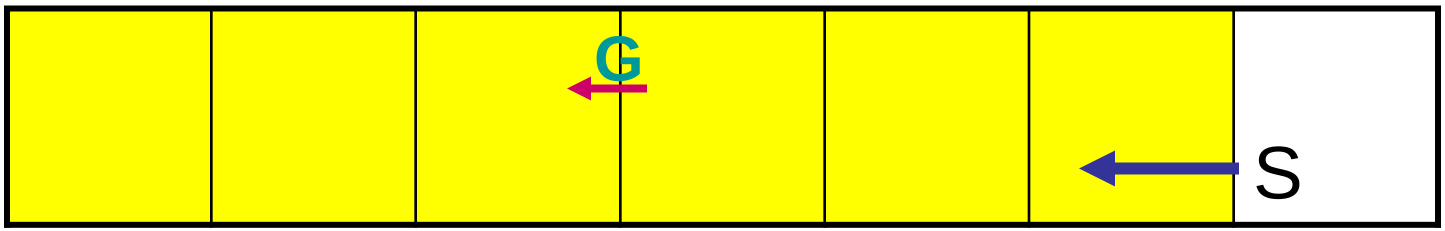
PCD_STiCM



$$v_x' = \frac{v_x + u_x}{1 + \frac{u_x v_x}{c^2}}$$

$$v_y' = v_y$$

$$v_z' = v_z$$



$$v_{relative} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{\left(4/5\right)c + \left(4/5\right)c}{1 + \frac{\left(4/5\right)c\left(4/5\right)c}{c^2}}$$

$$= \frac{\left(8/5\right)c}{1 + 16/25} = \frac{25}{41} \times \frac{8}{5} c = \frac{40}{41} c$$

Seeta clocks 3 years in her own clock and must shoot off toward Geeta at a speed of $\frac{40}{41}c$, and in subsequent 3-Seeta-yrs, catching up with Geeta.

Seeta clocks 3 years in her own clock and must shoot off toward Geeta at a speed of $\frac{40}{41}c$, and in exactly additional 3-Seeta-yrs, (i.e., as per Seeta's clock) she must catch up with Geeta.

Will this work?

Constraints: While all this happens, Seeta must clock 3+3=6 yrs in *her* clock, and Geeta must clock 10 yrs in *her own* clock.

For how many 'home-bound clock years' must Geeta travel (from Seeta's perspective) so that she (G) finds, that *as per her own (Geeta's) clock*, she has aged by 10 years?

$$\Delta\tau = 10\, years$$

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}; \ \beta = \text{v/c} = \frac{4}{5} = 0.8$$

$$10 = \Delta\tau = \Delta t\sqrt{1-\beta^2} = \Delta t\sqrt{1-(0.8)^2} = \Delta t\sqrt{0.36} = \Delta t \times 0.6$$

$$\Delta t = \frac{10}{0.6} = 16.6667\, yrs \text{ in units of home-bound clock.}$$

How much distance would Geeta travel over this period? $\text{distance} = \text{speed} \times \text{time}$

$$\text{d} = (0.8c) \times 16.66667 \ = 0.8 \times \left(c \ in \ \frac{ly}{yr} \right) \times 16.66667\, yrs = 13.333336 \ ly$$

$$d = (0.8c)\times 16.66667 \ = 0.8\times\left(c \ in \ \frac{ly}{yr}\right)\times 16.66667\,yrs = 13.333336 \ ly$$

Now, after 3 years of the home-bound clock, Seeta starts off to cover that distance -

We have estimated already that Seeta would travel at

a speed of $\left(\frac{40}{41}\right)c$

How much distance must Seeta now travel to catch up with Geeta?

$13.333336 \ ly$?

This distance, for Seeta, must look Lorentz-contracted!

The Lorentz-contracted distance Seeta would need to travel to catch up with Geeta is:

$$c : \frac{ly}{yr}$$

$$d' = d\sqrt{1-\beta^2} = 13.333336 \times \sqrt{1-\left(\frac{40}{41}\right)^2} = 13.333336 \times \sqrt{\frac{1681-1600}{1681}}$$

$$d' = 13.333336 \times \frac{9}{41} = 2.926829\ ly$$

How much time will Seeta take to travel this distance at the speed $\left(\frac{40}{41}\right)c$?

$$time = \frac{\text{distance}}{\text{speed}} = \frac{2.926829\ ly}{\frac{40}{41} \times 1\ \frac{ly}{yr}} = 2.926829 \times \frac{41}{40} = 3\ years$$

Again, ***Seeta ages by 3+3 of her clock's years while Geeta ages by 10 years*** ……. No paradox!

Symmetry/Equivalence principle in STR:

No matter whose perspective we consider, it is Geeta who must age by 10 years and Seeta by 6 years.

We see that in either case, ***Seeta ages by 3+3 of her clock's years while Geeta ages by 10 years of her own clock years***……. No paradox!

…… but then,

in the final analysis,

why do our observers have to be 'twins' ?

Time Dilation for Particles

Excited states: have an ‘intrinsic clock’ that determines the half-life of a decay process.

Rate at which the ‘intrinsic clock’ ticks in a moving frame, as observed by a static observer, is slower than the rate of a static clock.

‘half-life’ of a moving particles appears, to the static observer, to be increased.

We will take a Break…

…… Any questions ?

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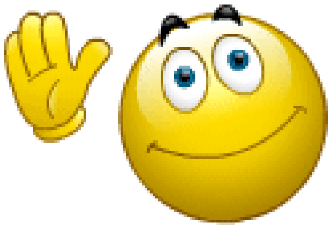
$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}; \quad \beta = \mathrm{v/c} \langle 1 \;; \quad L' = l\sqrt{1\text{-}\beta^2} \le l$$



Next L22 :

STR – conclusions

Mass-Energy equivalence,

STR+QM →electron spin,

Mass / Gravity? / GTR

STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 22

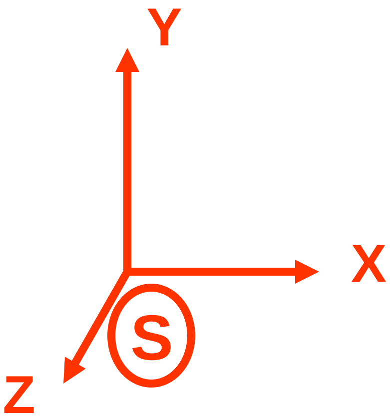
$$\Delta t = \frac{\Delta \tau}{\sqrt{1-\beta^2}}; \quad \beta = \text{v/c} \langle 1 \; ; \quad L' = l\sqrt{1\text{-}\beta^2} \le l$$

Unit 6 : Special Theory of relativity - conclusions

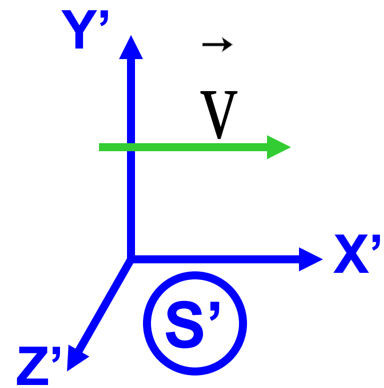
mass-energy equivalence
STR+QM→electron spin
Mass / Gravity ? / GTR

First few Nobel Prizes in Physics, in reverse chronological order

1922 - Niels Bohr
1921 - Albert Einstein
1920 - Charles Edouard Guillaume
1919 - Johannes Stark
1918 - Max Planck
1917 - Charles Glover Barkla
1916 - The prize money was allocated to the Special Fund of this prize section
1915 - William Bragg, Lawrence Bragg
1914 - Max von Laue
1913 - Heike Kamerlingh Onnes
1912 - Gustaf Dalén
1911 - Wilhelm Wien
1910 - Johannes Diderik van der Waals
1909 - Guglielmo Marconi, Ferdinand Braun
1908 - Gabriel Lippmann
1907 - Albert A. Michelson
1906 - J.J. Thomson
1905 - Philipp Lenard
1904 - Lord Rayleigh
1903 - Henri Becquerel, Pierre Curie, Marie Curie
1902 - Hendrik A. Lorentz, Pieter Zeeman
1901 - Wilhelm Conrad Röntgen



Origins O and O' of the two frames S and S' coincide at t=0 and t'=0.



$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$

$Note: \gamma \to 1$ as $v \to 0$.

Lorents transformations transform the space-time coordinates of ONE EVENT.

What is SPACE for one observer, is a *mix* of space and time for another !

What is TIME for one observer, is a *mix* of time and space for another !

Now, static charges produce electric fields.

Current (charge in motion) produces a magnetic field.

$$E'_x = E_x$$

$$E'_y = \gamma_f \left[E_y - \mathrm{v}_f B_z \right]$$

$$E'_z = \gamma_f \left[E_z - \mathrm{v}_f B_y \right]$$

STR: What is $\vec{E}$ or $\vec{B}$ for one observer, is a ***mix*** of $\left(\vec{E} \text{ and } \vec{B}\right)$ for another !

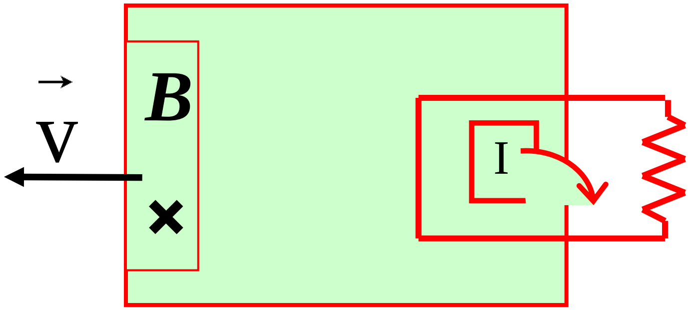
$$B'_x = B_x$$

$$B'_y = \gamma_f \left[B_y + \frac{\mathrm{v}_f}{c^2} E_z \right]$$

$$B'_z = \gamma_f \left[B_z + \frac{\mathrm{v}_f}{c^2} E_y \right]$$

Faraday-Lenz experiments now make sense!

Faraday's experiments

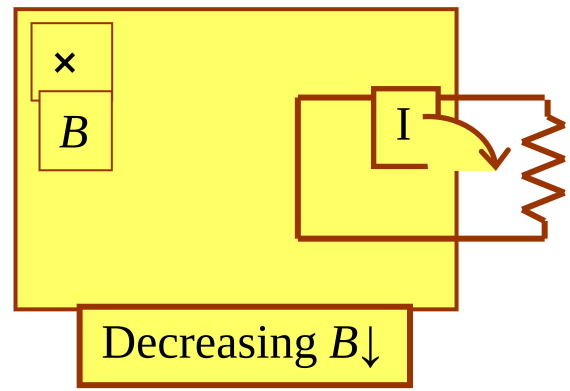


Loop held fixed; Magnetic field dragged toward left.

NO Lorentz force $q\left(\vec{v}\times\vec{B}\right)$

Current: identical!

Strength of B *decreased*.
Nothing is moving,
but still, current seen!!!



$$I \propto \frac{dB}{dt}$$

Einstein:

Special Theory of Relativity

P. Chaitanya Das, G. Srinivasa Murty,

K. Satish Kumar, T A. Venkatesh

and P.C. Deshmukh

'Motion of Charged Particles in Electromagnetic Fields and Special Theory of Relativity'

Resonance, Vol. 9, Number 7, 77-85 (2004)

http://www.ias.ac.in/resonance/July2004/pdf/July2004Classroom3.pdf

Other implications of STR: space-time continuum

INVARIANT INTERVALS?

$$\vec{\eta} = \frac{d\vec{r}}{d\tau} = \frac{d\vec{r}}{dt/\gamma} = \gamma\frac{d\vec{r}}{dt} = \gamma\vec{\mathrm{v}}$$

$$\text{'velocity'} \xrightarrow{?} \frac{d\vec{r}}{dt}$$

$$\eta^{\mu} = \{\gamma c, \gamma\vec{\mathrm{v}}\} : \text{"4-velocity"}$$

$$\eta^{\mu} = \{\eta^{0}, \eta^{1}, \eta^{2}, \eta^{3}\} = \{\gamma c, \gamma\vec{\mathrm{v}}\}$$

$$\eta^{0}\eta_{0} + \eta^{1}\eta_{1} + \eta^{2}\eta_{2} + \eta^{3}\eta_{3} = c^{2}$$ Lorentz Invariant

$$\eta^0\eta_0 + \eta^1\eta_1 + \eta^2\eta_2 + \eta^3\eta_3 = c^2$$

$$\vec{\eta} = \frac{d\vec{r}}{d\tau} = \gamma\vec{\mathrm{v}} \qquad \vec{p} = m\vec{\eta}$$

$$p^\mu = \left\{p^0, p^1, p^2, p^3\right\} = \left\{\gamma mc, \gamma m\vec{\mathrm{v}}\right\} = \left\{\frac{E}{c}, \gamma m\vec{\mathrm{v}}\right\}$$

$$p^0p_0 + p^1p_1 + p^2p_2 + p^3p_3 = \frac{E^2}{c^2} - \vec{p}\bullet\vec{p}$$

$$\frac{E}{c} = \gamma mc$$

$$= m^2c^2$$

$$E^2 = p^2c^2 + m^2c^4$$

For a photon

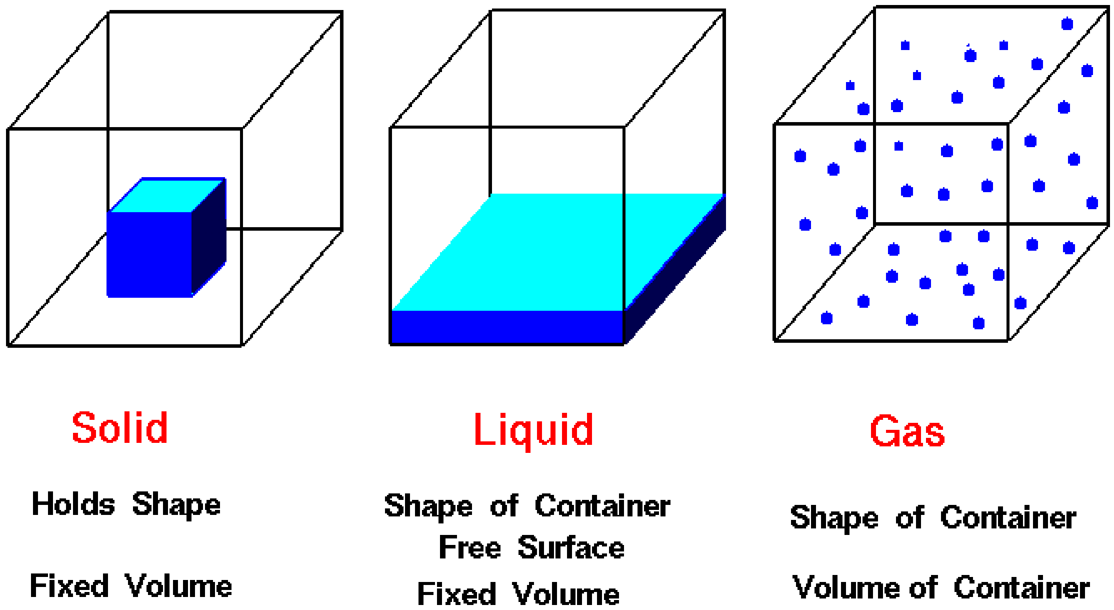
$$\mathrm{v} = c$$

$$E = pc$$

$$E_{rest} = \gamma mc^2; \quad \Rightarrow E_{rest} = \frac{mc^2}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}$$

Questions remain!

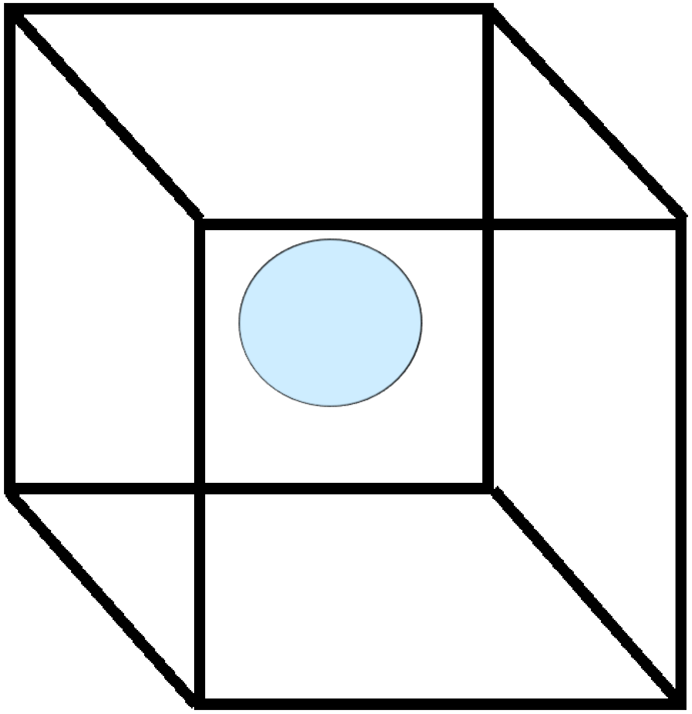
What’s GRAVITY?

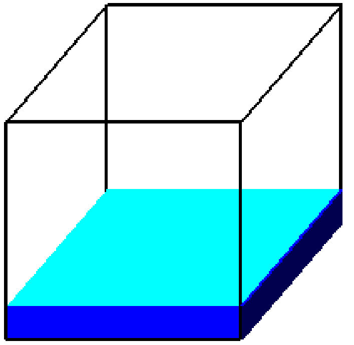


☺ What will be the shape of a tiny little amount of a liquid in a closed (sealed) beaker when this ‘**liquid-in-a-beaker**’ system is

(a) on earth

(b) orbiting in a satellite around the earth.



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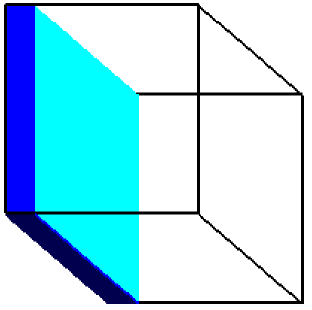


Gravity - Geometry

The curvature of space-time continuum reproduces the effects that we normally attribute to the gravitational interaction.

The space-time curvature of space-time itself is determined by the presence of matter!

Mass causes the space-time to acquire such a curvature that other matter is attracted toward it…. which is what we have referred to as gravitational attraction!

Einstein's General Theory of Relativity 1915 ….. Field Equations of GTR

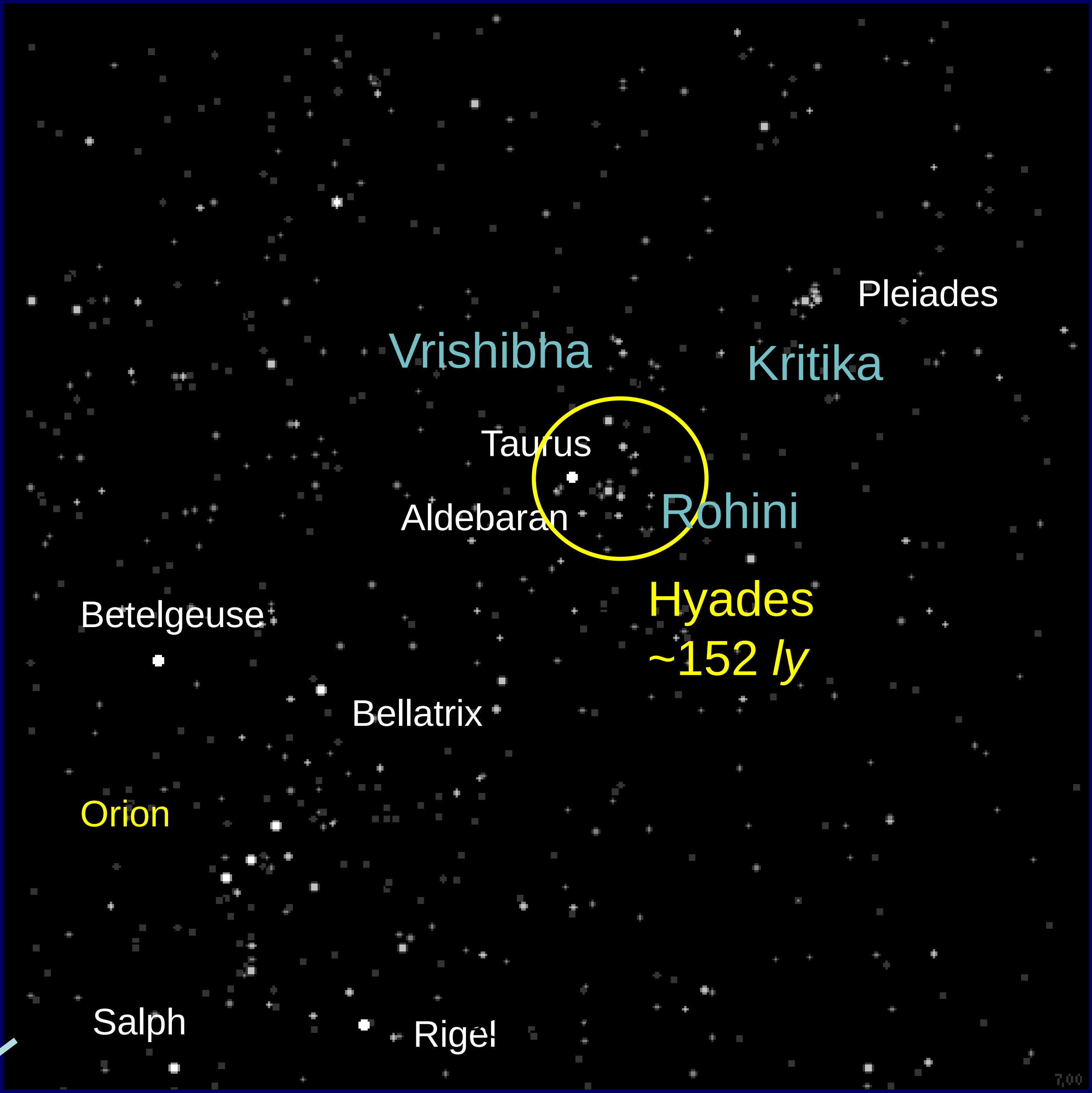
Eddington’s experiments

Total eclipse of 29 May 1919.

During the period of the total eclipse,

the Sun would be right in front of the

Hyades, a cluster of bright stars.



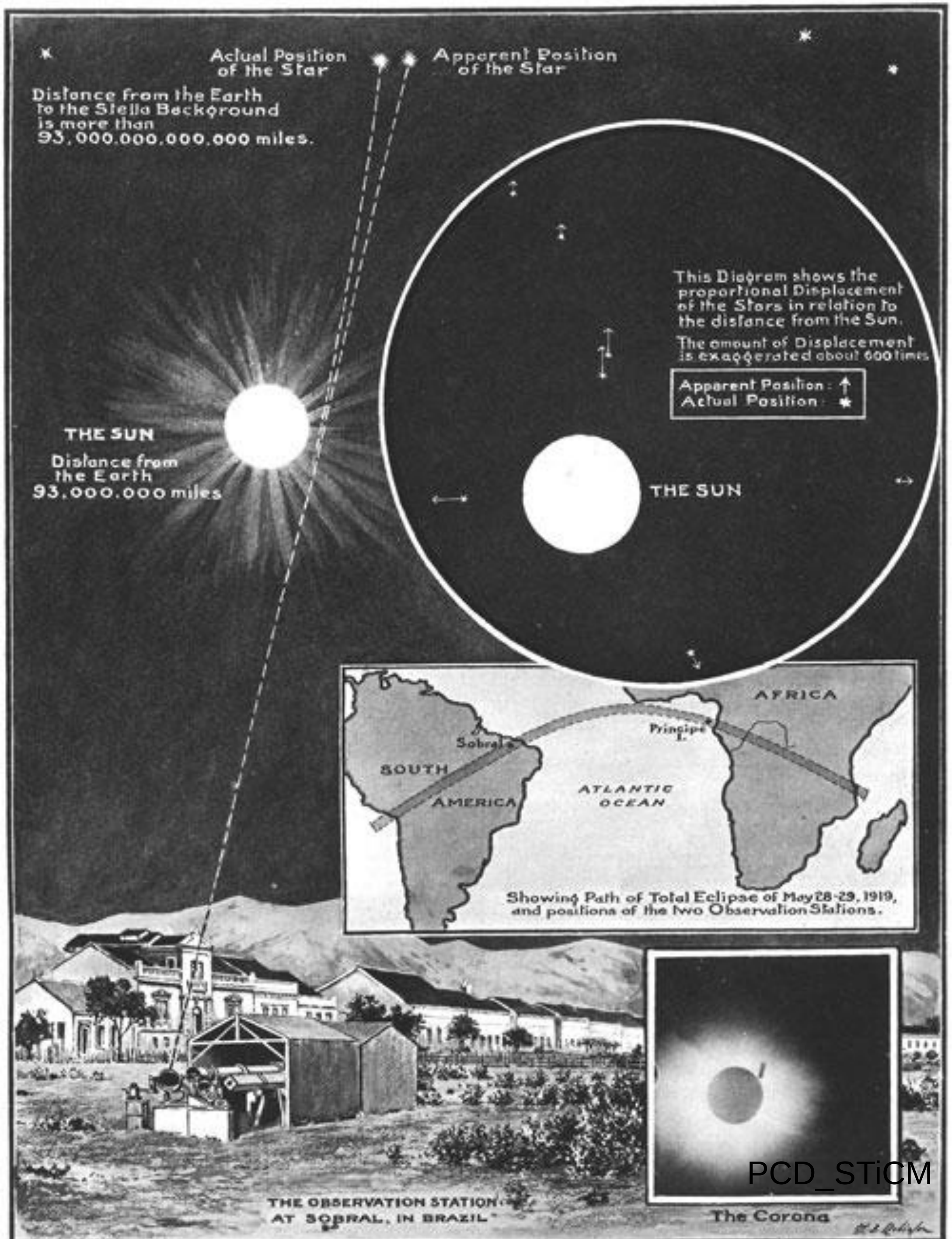
Vyadha

Sirius

F. W. Dyson, A. S. Eddington, and C. Davidson, "A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919"

Philosophical Transactions of the Royal Society of London. Series A, (1920): 291-333, on 332.

GTR predicted twice as much deflection of light rays passing the Sun as did STR.

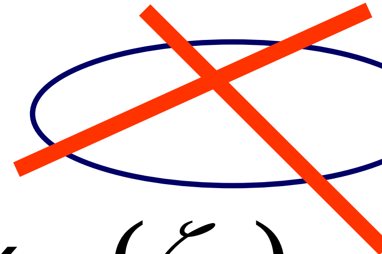


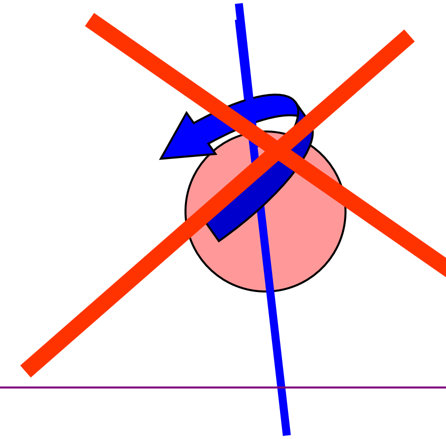
Arthur Eddington

http://denisdutton.com/einstein_eddington.htm

Observation station at
SOBRAL, BRAZIL

SPIN-ORBITALS





$$u_i(q_j) = u_{n_i, l_i, m_{l_i}}(\vec{r}_j)\, \chi_{m_{s_i}}(\zeta_j)$$

$$\vec{s} \times \vec{s} = i\hbar \vec{s}; \qquad [s_x, s_y] = i\hbar s_z$$

$$s^2 \,|\, s, m_s \rangle = \hbar^2 s(s+1) \,|\, s, m_s \rangle$$

$$s_z \,|\, s, m_s \rangle = \hbar m_s \,|\, s, m_s \rangle$$

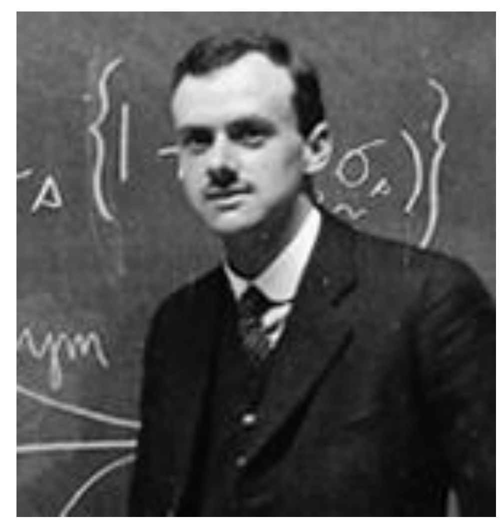
$s = \frac{1}{2}$: fixed internal property of an electron

$$m_s = (-s, \ldots., s) = -\frac{1}{2}, +\frac{1}{2}$$

1928: Dirac STR+QM

Relativistic Quantum Mechanics

Provided formal basis for electron's spin



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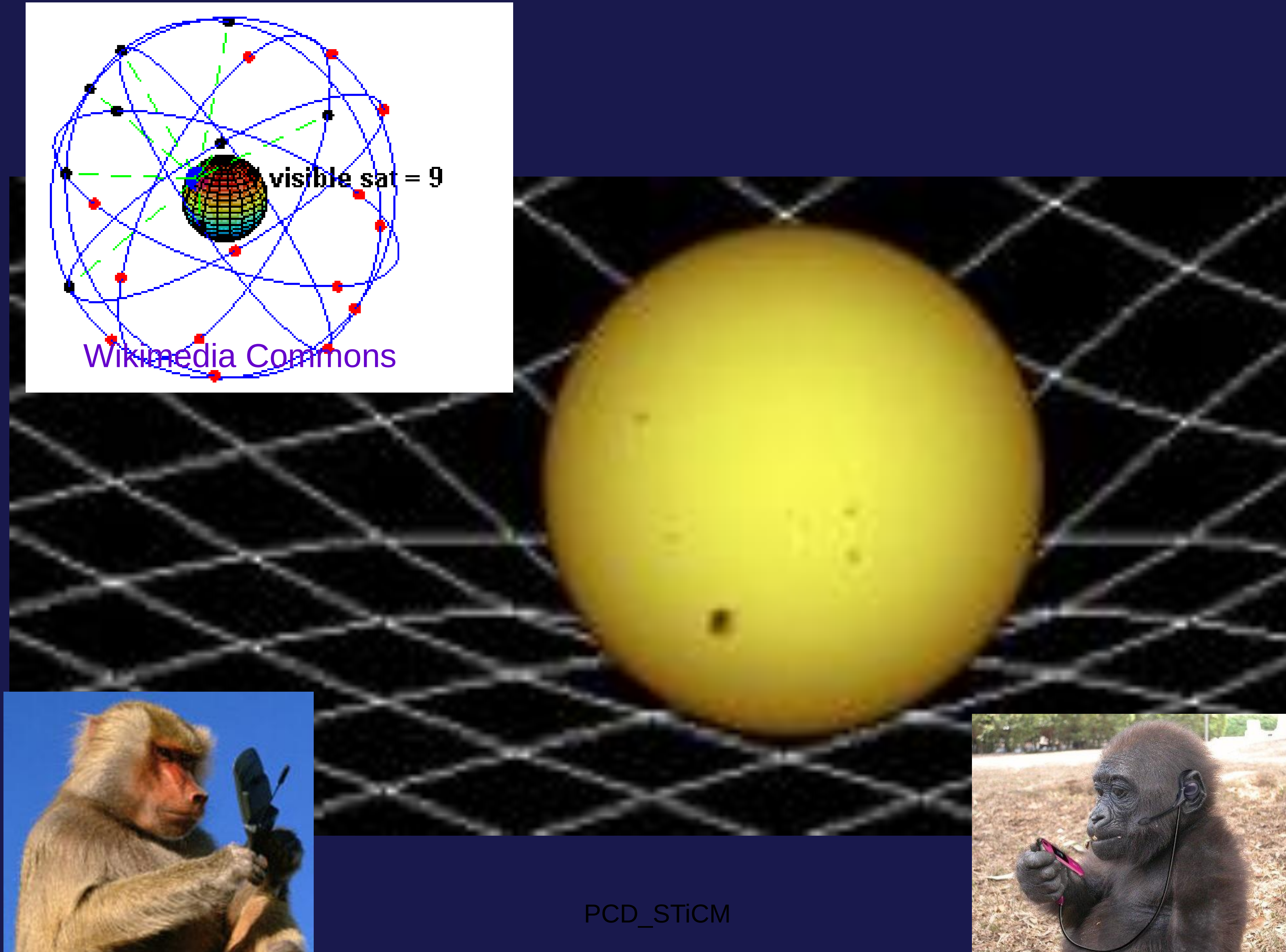
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Wikimedia Commons

Is

Newtonian / Lagrangian / Hamiltonian Mechanics

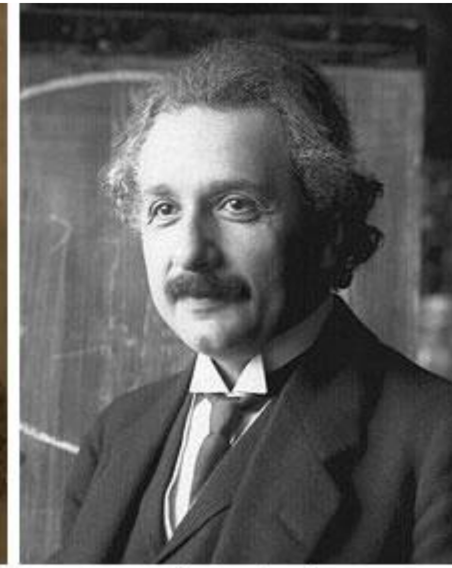
Wrong?

Is Galilean Relativity Wrong?

$$\frac{\mathrm{v}}{c} \langle \langle 1 \; ; \qquad \mathrm{v} \to 0 \; ; \hbar \to 0$$

We conclude the unit 6 with a quote from Albert Einstein:

Satyendra Nath Bose



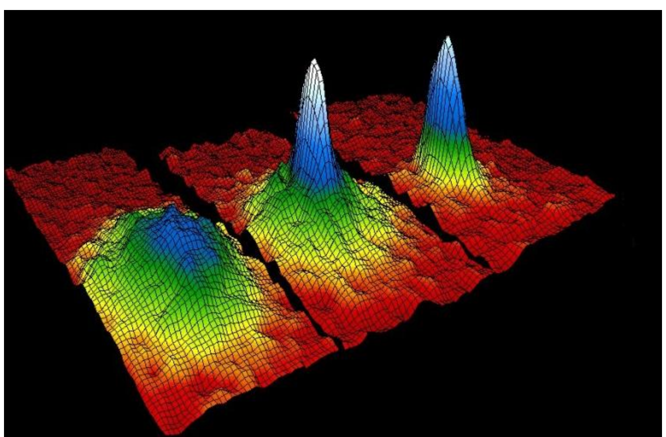
Albert Einstein

If at first the idea is not absurd, then there is no hope for it

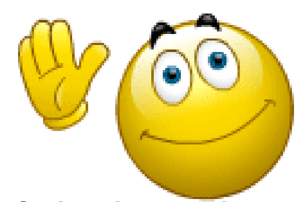
- Albert Einstein

-No guarantee that there is hope for every absurd idea!

- Our experience !!!



Next L23 : Unit 7
Potentials,
Gradients, Fields



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http://www.physics.iitm.ac.in/~labs/amp/

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